Name: **ID:**

Midterm #3 - P3A - Version A

Prof. Laszlo Bardoczi, December 1, 2025

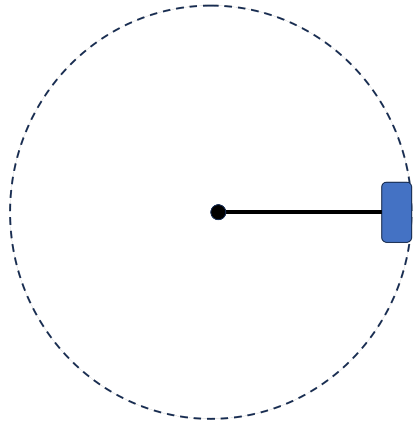
- The exam is 40 minutes long and contains 8 calculation problems (1-2 steps each).
- You must solve 6 problems of your choice; 2 problems will not be graded.
- Each problem is worth up to 1 point: 0.5 point for the correct numerical value (to 2 significant figures) and 0.5 point for correct SI units. An incorrect sign results in a -0.5 point penalty. A correct unit with an incorrect numerical value receives no credit.
- Calculations are broken into 1-2 step individual problems to award "partial credit". Such credits are based solely on the boxed answers.
- Write final answers clearly in the designated boxes. Leave the boxes empty for the problems you do not wish to be graded. If all boxes are filled, Problems #7-8 will not be graded.
- A detachable formula sheet is provided at the end of the exam.
- At the end of the exam, submit only the front page, present your UCI ID for verification and sign in.
- Direct all grading questions to your TA.
- Cheating will result in an automatic failing grade for the course.

Q#	Answer	Q#	Answer
1		2	
3		4	
5		6	
7		8	

Problem 1

A 0.10 kg biology sample is attached to the end of a 0.20 m horizontal centrifuge arm. The arm rotates in a horizontal circle at constant speed, as shown in the top-down diagram. The only horizontal force pulling the sample toward the center is the tension in the arm. During operation, this tension is 12 N.

Find the rotation frequency (f) of the centrifuge.



Solution:

For uniform circular motion, the inward (centripetal) force is

$$F_c = \frac{mv^2}{r}.$$

Here, the only inward force is the measured tension:

$$T = \frac{mv^2}{r}.$$

Solve for the speed:

$$v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{12 \times 0.20}{0.10}} = \sqrt{24} = 4.90 \text{ m/s}.$$

Uniform circular motion speed relates to frequency f by

$$v = 2\pi r f.$$

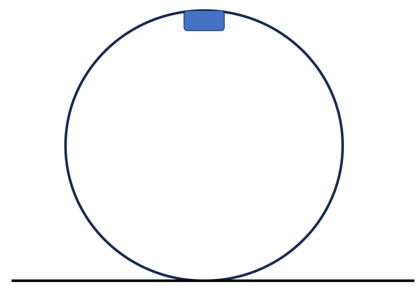
Thus:

$$f = \frac{v}{2\pi r} = \frac{4.90}{2\pi(0.20)} = \frac{4.90}{1.257} = 3.90 \text{ rev/s}.$$

Problem 2

A rollercoaster cart of mass 500 kg travels through a vertical circular loop of radius 12 m. At the very top of the loop, the cart is upside down. Assume the rail exerts only a normal force on the cart. Drag and friction are negligible.

What is the minimum velocity the cart must have at the top to maintain contact with the rail?



Solution:

At the top of the loop, the cart is upside down and the track is above it. To just maintain contact, the track must not need to push on the cart; that is, the normal force is zero:

$$N = 0.$$

In this limiting case, gravity alone supplies the centripetal force needed for the cart to follow the circular path. The cart "falls" toward the center of the loop at exactly the rate needed to stay on the curved track above it.

Thus:

$$mg = \frac{mv^2}{R}.$$

Mass cancels:

$$g = \frac{v^2}{R}.$$

Solve for the minimum speed:

$$v_{\min} = \sqrt{gR} = \sqrt{(9.8)(12)} = 10.8 \text{ m/s}.$$

Problem 3

A rod of mass 0.20 kg and length 1.2 m pivots about its center. Two identical point masses of 0.15 kg are attached symmetrically at a distance 0.40 m from the pivot. At the center is a solid cylindrical spool of radius 15 cm and mass 0.35 kg.

Find the moment of inertia of the system.

Solution:

Moment of inertia of the rod:

$$I_{\text{rod}} = \frac{1}{12}ML^2 = \frac{1}{12}(0.20)(1.2)^2 = 0.024 \text{ kg m}^2.$$

Moment of inertia of the two point masses:

$$I_{\text{masses}} = 2mx^2 = 2(0.15)(0.40)^2 = 0.048 \text{ kg m}^2.$$

Solid cylindrical spool at the center

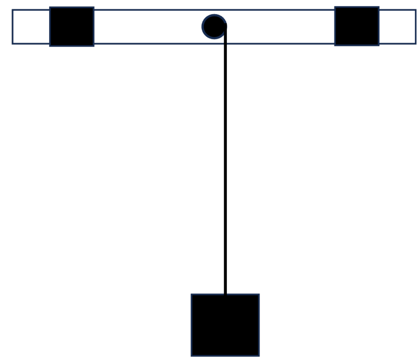
$$I_{\text{spool}} = \frac{1}{2}m_s R^2 = \frac{1}{2}(0.35)(0.15^2) = 0.00394 \text{ kg m}^2.$$

Total moment of inertia

$$I_{\text{total}} = 0.024 + 0.048 + 0.00394 = 0.07594 \text{ kg m}^2 \approx 7.6 \times 10^{-2} \text{ kg m}^2.$$

Problem 4

A rod of mass m_1 and length L_1 pivots about its center. Two identical point masses of mass m_2 are attached symmetrically at a distance L_2 from the pivot. The total moment of inertia of this rotating system is measured to be $I = 0.05\ \text{kg}\,\text{m}^2$. At the center is a massless cylindrical spool of radius 0.05 m with a rope wound around it. A hanging mass of 0.50 kg is attached to the rope. When released, the 0.50 kg mass falls and causes the rod–spool system to rotate without slipping.



Find the angular acceleration α of the rod.

Solution:

For the falling mass:

$$mg - T = ma, \qquad a = R\alpha,$$

Torque from the rope:

$$TR = I\alpha \quad \Rightarrow \quad T = \frac{I\alpha}{R}.$$

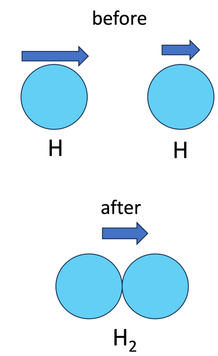
Substitute into the falling–mass equation:

$$mg - \frac{I\alpha}{R} = m(R\alpha).$$

Solve for α:

$$\alpha = \frac{mg}{mR + I/R} \approx 4.78\ \text{rad/s}^2.$$

Problem 5



Two hydrogen atoms move in the same direction along a straight line. Atom A moves with speed $v_1 = 900\,\mathrm{m/s}$ behind atom B that moves with speed $v_2 = 300\,\mathrm{m/s}$. They collide, bond, and form a single H_2 molecule.

Treat the event as a perfectly inelastic collision and find the final velocity of the H_2 molecule.

Solution:

This is a perfectly inelastic collision, so momentum is conserved:

$$mv_1 + mv_2 = (2m)v_f.$$

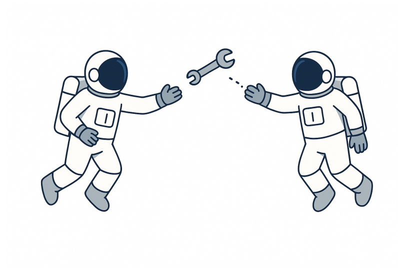
Solve for v_f:

$$v_f = \frac{v_1 + v_2}{2} = 600\,\mathrm{m/s}.$$

Problem 6

Two astronauts, each of mass 80 kg, are floating outside the ISS during a repair operation. Astronaut A is holding a 5 kg tool that Astronaut B needs. Astronaut A gently throws the tool toward Astronaut B, who catches it.

Assume the throw and the catch happen along a straight line in free space, with no external forces. If Astronaut A throws the tool at a speed of 2 m/s (relative to A at the moment of release), find the speed at which the two astronauts are separating *after* Astronaut B catches the tool.



Solution:

First, A throws the tool. Momentum before the throw is zero, and after the throw:

$$(5)(2) + (80)v_A = 0 \longrightarrow v_A = -0.125\,\text{m/s}.$$

I.e., A drifts backward at 0.125 m/s.

Next, B catches the tool. Just before the catch, the tool moves at 2 m/s, and B is at rest. Momentum before catch:

$$p_i = (5)(2) + (80)(0) = 10.$$

After the catch, the combined mass is 85 kg:

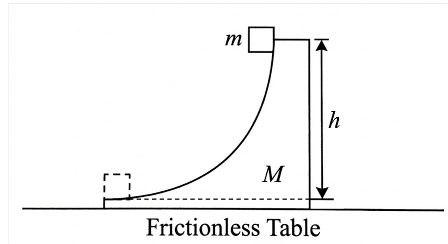
$$p_f = 85\,v_B \longrightarrow v_B = 0.118\,\text{m/s}$$

I.e. B drifts forward at 0.118 m/s after catching the tool.
The separation speed:

$$v_{\text{sep}} = v_B - v_A = 0.118 + 0.125 = 0.243\,\text{m/s}.$$

Problem 7

A frictionless wedge of mass 1 kg rests on a horizontal, frictionless table and is free to slide. A small block of mass 0.20 kg is placed at the top of the wedge at height $h = 10\,\text{cm}$ and released from rest. The wedge has a curved (circular) shape such that the block leaves the wedge *horizontally* at the bottom.



Using conservation of momentum and energy, find the horizontal speed of the block the instant it leaves the wedge.

Solution:

Momentum conservation:

$$mv = MV \quad \Rightarrow \quad V = \frac{m}{M}v.$$

Energy conservation:

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}MV^2.$$

Substitute V:

$$mgh = \tfrac{1}{2}v^2\left(m + \frac{m^2}{M}\right).$$

Solve for v:

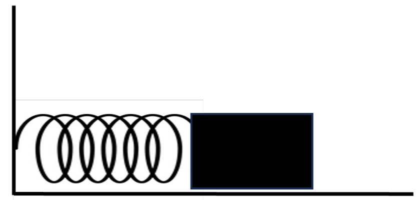
$$v^2 = \frac{2gh}{1 + \frac{m}{M}} \quad \longrightarrow \quad v = 1.28\ \text{m/s}\,.$$

Note: If the wedge is fixed to the ground (so it cannot recoil), then effectively $M \to \infty$ and the term m/M vanishes. In this limit, the expression reduces to the familiar free–fall result:

$$v = \sqrt{2gh}.$$

Problem 8

Consider a simple harmonic oscillator made of a mass attached to a spring. When the mass is halfway between its equilibrium position and the farthest point it reaches during the motion (i.e., halfway to maximum stretch or compression), what is the ratio of its kinetic energy to its spring potential energy (K/U)?



Solution:

As the oscillator moves, energy continuously shifts between potential and kinetic. At the endpoints ($x = A$), the velocity is zero, so all the mechanical energy is stored as spring potential energy:

$$E = \tfrac{1}{2}kA^2.$$

Spring potential energy at $x = \frac{1}{2}A$:

$$U = \frac{1}{2}kx^2 = \tfrac{1}{2}k\left(\tfrac{1}{2}A\right)^2 = \tfrac{1}{4}E.$$

So the kinetic energy there is:

$$K = E - U = \tfrac{3}{4}E.$$

Thus the ratio is 3.

Physics 3A Formula Sheet

Vectors

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$
$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$
$$A_x = |\vec{A}|\cos\theta$$
$$A_y = |\vec{A}|\sin\theta$$
$$\tan\theta = \frac{A_y}{A_x}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j}$$
$$\vec{A} + \vec{B} = (A_x + B_x, A_y + B_y)$$
$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = |\vec{A}||\vec{B}|\cos\phi$$

Trigonometry

$$\sin\theta = \cos(90^\circ - \theta)$$
$$\cos\theta = \sin(90^\circ - \theta)$$
$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$F_x = |\vec{F}|\cos\theta$$
$$F_y = |\vec{F}|\sin\theta$$

Geometry Essentials

$$c^2 = a^2 + b^2$$
$$C = 2\pi r; A = \pi r^2; s = r\theta$$
$$A = 4\pi r^2; V = \frac{4}{3}\pi r^3$$

Diffusion and Proportional Reasoning

$$r_{\text{rms}} = d\sqrt{mn}$$
$$r_{\text{rms}}^2 = d^2mn$$
$$D = \tfrac{1}{2}vd$$
$$r_{\text{rms}} = \sqrt{2mDt}$$

$$y = Cx^n$$
$$\frac{y_2}{y_1} = \frac{Cx_2^n}{Cx_1^n}$$

Kinematics

$$\Delta x = v\Delta t$$
$$v_{\text{avg.}} = \Delta x/\Delta t$$
$$a = \Delta v/\Delta t$$
$$x(t) = x_0 + vt$$
$$v(t) = v_0 + at$$
$$x(t) = x_0 + v_0t + \frac{1}{2}at^2$$
$$x(t) = \int v(t)\,dt$$
$$v(t) = \int a(t)\,dt$$

$$v(t) = \frac{dx}{dt}$$
$$a(t) = \frac{dv}{dt}$$
$$\frac{d}{dt}[t^n] = nt^{n-1}$$
$$\int_{t_1}^{t_2} t^n\,dt = \frac{t_2^{n+1} - t_1^{n+1}}{n+1}$$
$$y(t) = v_{y0}t - \frac{1}{2}gt^2$$
$$t_{\text{flight}} = \frac{2v_{y0}}{g}$$

$$y_{\max} = \frac{v_{y0}^2}{2g}$$
$$x_{\text{final}} = \frac{2v_{x0}v_{y0}}{g}$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x_{\text{end}} = \frac{v_{x0}}{g}\left(v_{y0} + \sqrt{v_{y0}^2 - 2gy_{\text{end}}}\right)$$
$$t_{\text{flight}} = \frac{v_{y0} + \sqrt{v_{y0}^2 - 2gy_{\text{end}}}}{g}$$
$$v_y = -\sqrt{v_{y0}^2 - 2gy_{\text{end}}}$$

Interacting Systems

$$F_g = mg$$
$$F_s = -k\,\Delta x$$
$$K = k_1 + k_2$$
$$\frac{1}{K} = \frac{1}{k_1} + \frac{1}{k_2}$$
$$F_T = \text{constant along the rope}$$
$$F_N = mg\cos\theta$$
$$F_{\mu,s} \leq \mu_s N$$
$$F_{\mu,k} = \mu_k N$$
$$F_D = -\tfrac{1}{2}C_d\,\rho A v^2$$
$$F_D = -6\pi\,\eta\,r\,v$$
$$F_{\text{thrust}} = \dot{m}\,v_{\text{exhaust}}$$
$$a = \frac{F_{\text{net}}}{m}$$

Circular Motion, Rotation

$$\tau = rF\sin(\theta)$$
$$\tau = I\alpha$$
$$f = \frac{1}{T}$$
$$v = \frac{2\pi r}{T} = 2\pi r f$$
$$\omega = 2\pi f$$
$$\omega = \omega_\circ + \alpha t$$
$$\phi = \omega t$$
$$\phi = \omega t + \frac{1}{2}\alpha t^2$$
$$a_{\text{cp}} = \frac{v^2}{r}$$
$$F_{\text{cp}} = m\,\frac{v^2}{r}$$
$$I = mr^2$$
$$I = \tfrac{1}{2}mR^2$$
$$I = \tfrac{2}{5}mR^2$$
$$I = \tfrac{2}{3}mR^2$$
$$I = \tfrac{1}{12}mL^2$$
$$I = \tfrac{1}{3}mL^2$$

Momentum

$$p = mv$$
$$dp/dt = F(t)$$
$$p_f - p_i = J$$
$$\Delta p_1 + \Delta p_2 = 0$$
$$(v_1)_f = \frac{m_1 - m_2}{m_1 + m_2}\,(v_1)_i$$
$$(v_2)_f = \frac{2m_1}{m_1 + m_2}\,(v_1)_i$$

Energy

$$K = \tfrac{1}{2}mv^2$$
$$U = mgh$$
$$U = \tfrac{1}{2}kx^2$$
$$E = K + U$$
$$W = Fx$$
$$W = \int F(x)dx$$
$$W = \Delta K$$
$$W = \Delta E$$
$$P = \frac{dW}{dt} = Fv$$