Physics 3A: Physics for the Life Sciences

Chapter 8: Momentum

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Looking Ahead

Impulse
A golf club delivers an impulse to the ball during the strike. The size of the impulse determines the ball's momentum as it flies away.

> You'll learn that a **longer-lasting, stronger force** delivers a **larger impulse**.

Conservation of Momentum
A heavy curling stone keeps moving almost steadily because it has a large amount of momentum—a strong tendency to keep going.

> You'll see that the **total momentum of an isolated system is conserved**.

Angular Momentum
A spinning diver has *angular* momentum—a tendency to keep rotating.

> You'll learn a new **before-and-after problem-solving strategy** using conservation of linear and angular momentum.

Goal
To understand impulse, momentum, angular momentum, and how conservation laws provide powerful tools for solving problems.

Learning Objectives

Calculate the momentum of an object and the impulse exerted on it.

Identify isolated systems and apply momentum conservation.

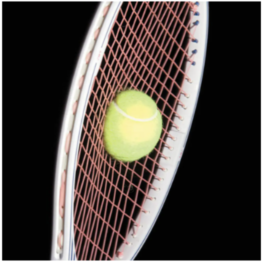
Apply the law of conservation of momentum to **collisions** and **explosions**.

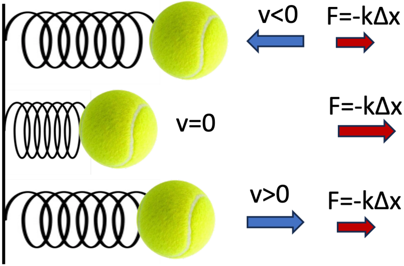
Understand and use **angular momentum** and its **conservation law**.

Sudden interactions, called collisions, involve large forces acting for very short times.

During a collision, objects compress and then re-expand (much like a spring). The force varies in time, often rising to a peak at maximum compression.

The force of the collision is zero before contact, increases rapidly during compression, then drops back to zero after separation. We describe this force as $F_x(t)$.

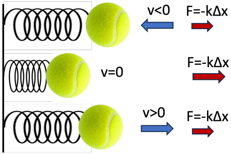


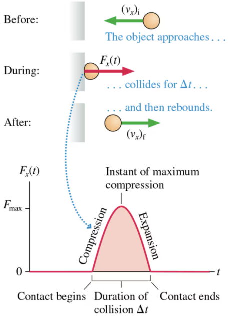


Using Newton's second law in 1D:

$$F_x(t) = ma_x = m\frac{dv_x}{dt} = \frac{d}{dt}(mv_x) = \boxed{\frac{d}{dt}p_x = F_x(t)}$$

where $\boxed{mv_x = p_x}$ is the momentum.





Multiplying both sides by dt and integrating:

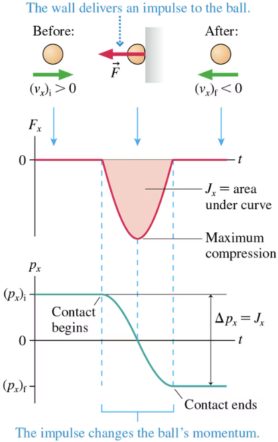
$$\int_{p_i}^{p_f} dp_x = [p_x]_{p_i}^{p_f} = p_f - p_i = \int_{t_i}^{t_f} F_x(t)\, dt = J$$

This connects the change in momentum to the **area under the force–time graph**, called the impulse.
In short, the change of momentum is the impulse:

$$\boxed{p_f - p_i = J}$$

Momentum and Impulse

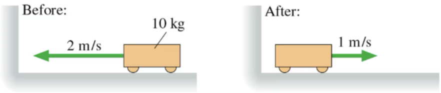
Figure 8.4 The momentum principle helps us understand a rubber ball bouncing off a wall.



Friendly reminder to complete your weekly reading assignment! The textbook has many excellent examples with clear, helpful explanations.

Stop to Think 8.1

The cart's change of momentum is



A. $-30 \text{ kg} \cdot \text{m/s}$
B. $-20 \text{ kg} \cdot \text{m/s}$
C. $0 \text{ kg} \cdot \text{m/s}$
D. $10 \text{ kg} \cdot \text{m/s}$
E. $20 \text{ kg} \cdot \text{m/s}$
F. $30 \text{ kg} \cdot \text{m/s}$

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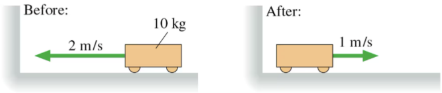

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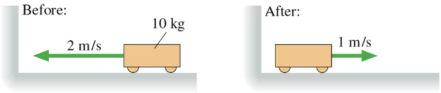
E. $20\ \mathrm{kg \cdot m/s}$
F. $30\ \mathrm{kg \cdot m/s}$

$p_i = -20kgm/s$, $p_f = 10kgm/s$, thus the change is $(p_f - p_i)$ 10-(-20)=30kgm/s

Stop to Think 8.1

The cart's change of momentum is

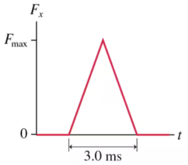


A. -30 kg · m/s
B. -20 kg · m/s
C. 0 kg · m/s
D. 10 kg · m/s
E. 20 kg · m/s
F. 30 kg · m/s

Example 8.1 **Hitting a baseball**

A 150 g baseball is thrown to the left with a speed of 20 m/s. It is hit straight back toward the pitcher at a speed of 40 m/s. The interaction force between the ball and the bat is shown in Figure 8.6. What *maximum* force F_{max} does the bat exert on the ball? What is the *average* force of the bat on the ball?

Figure 8.6 **The interaction force between the baseball and the bat.**



Momentum and Impulse

Example 8.1 **Hitting a baseball**

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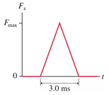
Figure 8.6 **The interaction force between the baseball and the bat.**



Recall: The change in momentum equals the impulse (the time integral of the force). Thus, the area of the triangular force–time graph (half times base times height),

$$J = \text{area} = \frac{1}{2} \Delta t \, F_{\text{max}},$$

must equal the momentum change,

$$\Delta p = (40\,\text{m/s} + 20\,\text{m/s})(0.15\,\text{kg}) = 9\,\text{Ns}.$$

Solving gives $F_{\text{max}} = 6000\,\text{N}$.
The average force is calculated from $F_{avg} \Delta t = J$, thus $F_{avg} = 3000 N$.

A spiny cushion

The spines of a hedgehog obviously help protect it from predators. But they serve another function as well. If a hedgehog falls from a tree—a not uncommon occurrence—it simply rolls itself into a ball before it lands. Its thick spines then cushion the blow by increasing the time it takes for the animal to come to rest. Indeed, hedgehogs have been observed to fall out of trees on purpose to get to the ground!

Momentum and Impulse

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The change in momentum (when free falling from height h) is

$$\Delta p = mv = m\sqrt{2gh}$$

During the collision, this must equal the impulse $J = \Delta t\, F_{avg}$. Thus, the average force is

$$F = \frac{m\sqrt{2gh}}{\Delta t}.$$

Momentum of a System

Momentum principle follows from Newton's 2nd law.

Total momentum of a system:

$$\boxed{P_{\text{tot}} = p_1 + p_2}$$

How can we see this? Consider two blocks with masses m_1 and m_2 that are stuck together and move with a common velocity v. Their individual momenta are

$$p_1 = m_1 v, \qquad p_2 = m_2 v.$$

The combined object has total momentum

$$P = (m_1 + m_2)v.$$

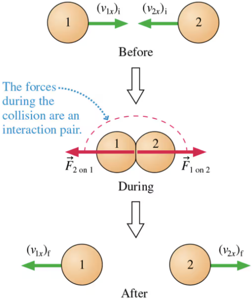
Thus,

$$p_1 + p_2 = P,$$

showing explicitly that the total momentum is the sum of the individual momenta.

Momentum Conserves During a Collisions

Consider two balls (1 and 2 in the figure) colliding with each other.



During a collision, the two objects exert forces on each other.

These forces are equal in magnitude and opposite in direction (Newton's 3rd law):

$$F_1 = -F_2$$

Because impulse is the time integral of force, the momentum changes are also equal and opposite:

$$\Delta p_1 = -\Delta p_2$$

Therefore, the total momentum remains unchanged (conserved) during the collision.

$$\Delta p_1 + \Delta p_2 = 0$$

Momentum Conservation

From Newton's 2nd law in momentum form:

$$\vec{F} = \frac{d\vec{p}}{dt}$$

For object 2:

$$(F_x)_{1 \text{ on } 2} = \frac{dp_{2x}}{dt}$$

For object 1:

$$(F_x)_{2 \text{ on } 1} = \frac{dp_{1x}}{dt}$$

Newton's 3rd law:

$$(F_x)_{1 \text{ on } 2} = -(F_x)_{2 \text{ on } 1}$$

Add the momentum equations:

$$\frac{d}{dt}(p_{1x} + p_{2x}) = 0$$

Therefore:

$$p_{1x} + p_{2x} = \text{constant}$$

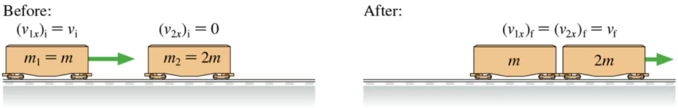
Same before and after collision:

$$(p_{1x})_f + (p_{2x})_f = (p_{1x})_i + (p_{2x})_i$$

Example: Two Colliding Train Cars

Initially, car 1 (mass m) moves with velocity v_i while car 2 (mass $2m$) is at rest. After the collision, they couple and move together with a common velocity v_f.

Figure 8.9 Two colliding train cars.



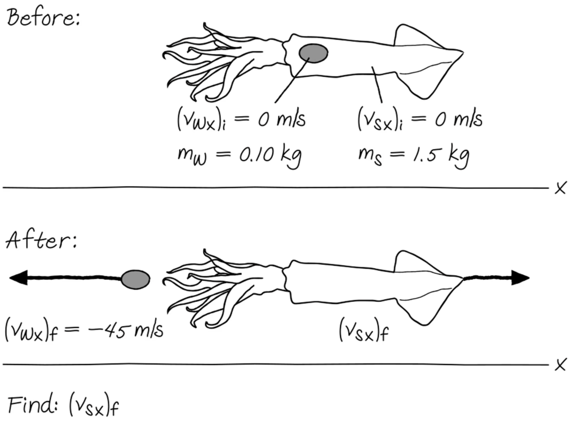
Momentum conservation ($p_f = p_i$) tells us the answer in one step:

$$mv_f + 2mv_f = mv_i + 0$$

$$3mv_f = mv_i \qquad \Rightarrow \qquad v_f = \frac{v_i}{3}$$

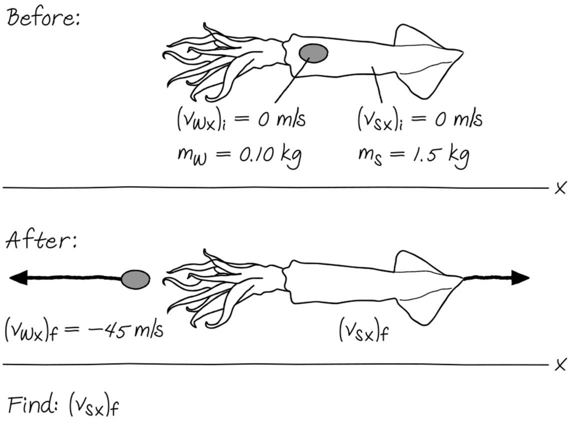
Example: Jetting Squid

Figure 8.10 **Before-and-after pictorial representation of a jetting squid.**



Example: Jetting Squid

Figure 8.10 **Before-and-after pictorial representation of a jetting squid.**

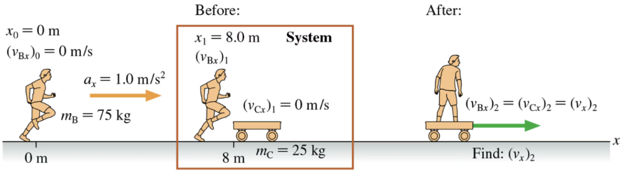


$$0 = p_1 = p_2 = -45 \cdot 0.1 + 1.5(v_{sx})_f \longrightarrow (v_{sx})_f = 45 \cdot 0.1/1.5 = 3$$

Bob Jumps on the Cart: Momentum in Action

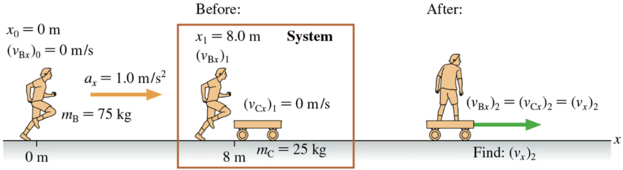
Bob accelerates for 8.0 m at $a_x = 1.0\,\mathrm{m/s^2}$, starting from rest.

Figure 8.11 Pictorial representation of Bob and the cart.



After reaching the cart, Bob jumps on. This is a **collision** between Bob and the cart, so we use **momentum conservation**.

Figure 8.11 **Pictorial representation of Bob and the cart.**



As before, the final velocity can be directly calculated from the acceleration and the distance:

$$(v_{Bx})_1 = \sqrt{2a_x x_1} = 4.0 \text{ m/s}$$

Before collision: Cart is at rest, so

$$p_{\text{initial}} = m_B (v_{Bx})_1 + 0$$

After collision: Bob and cart move together with common speed $(v_x)_2$:

$$(v_x)_2 = \frac{m_B}{m_B + m_C}(v_{Bx})_1 = \frac{75}{100} \times 4.0 = 3.0 \text{ m/s}$$

Key idea: Bob speeds up first (*kinematics*), then shares momentum with the cart (*momentum conservation*).

Elastic and Inelastic Collisions

Collisions are often divided into two categories: elastic and inelastic.
Elastic means: *able to return to its original shape after being stretched, compressed, or deformed.*
In physics, an elastic collision is one where no kinetic energy is lost ; objects bounce off each other without permanent deformation.

In all collisions, the **total momentum is conserved**:

$$m_1\vec{v}_{1,i} + m_2\vec{v}_{2,i} = m_1\vec{v}_{1,f} + m_2\vec{v}_{2,f}.$$

In summary:

Elastic Collision:

Momentum is conserved.
Kinetic energy is also conserved: objects "bounce off" without losing mechanical energy.

Inelastic Collision:

Momentum is conserved.
Kinetic energy is not conserved: some energy goes into heat, sound, or deformation.

A collision is called **perfectly inelastic** when objects stick together after impact (maximum loss of kinetic energy).

Example: Momentum Conservation in a Boat

Example. Two people sit in a small boat (initially at rest).

$$m_1 = 70 \text{ kg} \qquad m_2 = 60 \text{ kg} \qquad m_{\text{boat}} = 40 \text{ kg}$$

Person 1 jumps horizontally toward a dock with speed

$$v_1 = 3.0 \text{ m/s}$$

relative to the water.
Find the recoil speed of the boat and Person 2 immediately after the jump.

Solution.

The system (people + boat) has **no external horizontal forces**. Momentum is conserved:

$$0 = m_1 v_1 + (m_2 + m_{\text{boat}}) v_{\text{boat}}$$

Solve for the boat's recoil speed:

$$v_{\text{boat}} = -\frac{m_1}{m_2 + m_{\text{boat}}} v_1$$

Substitute numbers:

$$v_{\text{boat}} = -\frac{70}{60 + 40}(3.0) = -2.1 \text{ m/s}$$

The negative sign means the boat moves in the opposite direction of the jump.

Answer:

$$v_{\text{boat}} = 2.1 \text{ m/s (away from the dock)}$$

Elastic Collisions

We will learn about energy and energy conservation in Chapter 11.

For now, we just take the derived equations for velocities following elastic collisions when ball 2 is initially at rest:

$$(v_{1x})_f = \frac{m_1 - m_2}{m_1 + m_2}(v_{1x})_i \qquad (v_{2x})_f = \frac{2m_1}{m_1 + m_2}(v_{1x})_i$$

Three Special Elastic Collisions

Case a: $m_1 = m_2$

Ball 1 stops.
Ball 2 moves forward with $(v_{2x})_f = (v_{1x})_i$.
Equations give:

$$(v_{1x})_f = 0, \qquad (v_{2x})_f = (v_{1x})_i$$

Case b: $m_1 \gg m_2$

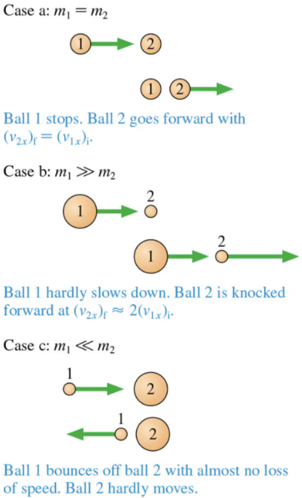
Ball 1 barely slows.
Ball 2 shoots forward at nearly $2(v_{1x})_i$.
In the limit $m_1 \to \infty$:

$$(v_{1x})_f \approx (v_{1x})_i, \qquad (v_{2x})_f \approx 2(v_{1x})_i$$

Case c: $m_1 \ll m_2$

Ball 1 bounces back.
Ball 2 barely moves.
In the limit $m_1 \to 0$:

$$(v_{1x})_f \approx -(v_{1x})_i, \qquad (v_{2x})_f \approx 0$$



Example: Elastic Collision

Example. Ball 1 of mass

$$m_1 = 0.40 \text{ kg}$$

moves to the right with speed

$$(v_{1x})_i = 5.0 \text{ m/s}.$$

It collides elastically with a stationary ball of mass

$$m_2 = 0.60 \text{ kg}.$$

Find the final velocities $(v_{1x})_f$ and $(v_{2x})_f$.

Solution.

Use the elastic-collision formulas (ball 2 initially at rest):

$$(v_{1x})_f = \frac{m_1 - m_2}{m_1 + m_2}(v_{1x})_i, \qquad (v_{2x})_f = \frac{2m_1}{m_1 + m_2}(v_{1x})_i.$$

Compute Ball 1's final speed:

$$(v_{1x})_f = \frac{0.40 - 0.60}{0.40 + 0.60}(5.0) = \frac{-0.20}{1.00}(5.0) = -1.0 \text{ m/s}.$$

Ball 1 bounces backward.

Compute Ball 2's final speed:

$$(v_{2x})_f = \frac{2(0.40)}{1.00}(5.0) = 4.0 \text{ m/s}.$$

Ball 2 moves forward.

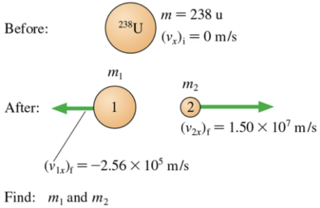
Explosions

An **explosion** is the opposite of a collision: pieces move apart after a brief, intense internal interaction.

Explosive forces (expanding gases, nuclear forces, springs) are **internal**.

If the system is isolated, **total momentum is conserved** during an explosion.

Example: radioactive decay of a nucleus into two fragments.



An explosion is mathematically like an inelastic collision run backward in time, where the internal energy source restores or increases kinetic energy instead of dissipating it. In this case kinetic energy increase comes from nuclear potential energy.

Example: Radioactive Decay (Explosion)

A $^{238}\mathrm{U}$ **nucleus** (initially at rest) disintegrates into:

a daughter nucleus of mass m_1, recoil speed $|v_{1x}| = 2.56 \times 10^5\ \mathrm{m/s}$ (to the left)

an ejected fragment of mass m_2, speed $v_{2x} = 1.50 \times 10^7\ \mathrm{m/s}$ (to the right)

Find: masses m_1 and m_2 given the total mass is $238\ \mathrm{u}$.

Momentum Conservation (system initially at rest):

$$m_1 v_{1x} + m_2 v_{2x} = 0$$

$$m_1(2.56 \times 10^5) = m_2(1.50 \times 10^7)$$

$$\frac{m_1}{m_2} = \frac{1.50 \times 10^7}{2.56 \times 10^5} \approx 58.6$$

Mass constraint:

$$m_1 + m_2 = 238\ \mathrm{u}$$

$$58.6\, m_2 + m_2 = 238 \quad \Rightarrow \quad m_2 \approx 4.0\ \mathrm{u}$$

$$m_1 = 238 - m_2 \approx 234\ \mathrm{u}$$

Final Answer:

$$m_1 \approx 234\ \mathrm{u}, \qquad m_2 \approx 4.0\ \mathrm{u}$$

Definition of Angular Momentum

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

Magnitude:

$$L = rp \sin\theta$$

Time Rate of Change: Torque

$$\frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r} \times \vec{F}$$

Conservation Law

If the **net external torque is zero**,

$$\frac{d\vec{L}}{dt} = 0 \qquad \Rightarrow \qquad \vec{L} = \text{constant}$$

Applies to point particles and rigid bodies.

Examples:

A spinning figure skater pulling arms in $\rightarrow$ faster spin.
Planetary motion around the Sun.

Linear vs. Angular Quantities

Linear (Translational) Motion

Momentum

$$\vec{p} = m\vec{v}$$

Newton's Second Law

$$\vec{F} = m\vec{a} \qquad \text{and} \qquad \vec{F} = \frac{d\vec{p}}{dt}$$

Conservation

$$\text{If } \sum \vec{F}_{\text{ext}} = 0 \quad \Rightarrow \quad \vec{p} = \text{constant}$$

Rotational (Angular) Motion

Angular Momentum

$$L = I\omega$$

Rotational Newton's Law

$$\vec{\tau} = I\vec{\alpha} \qquad \text{and} \qquad \vec{\tau} = \frac{d\vec{L}}{dt}$$

Conservation

$$\text{If } \sum \vec{\tau}_{\text{ext}} = 0 \quad \Rightarrow \quad \vec{L} = \text{constant}$$

Example: Conserving Angular Momentum

Example. An ice skater is spinning with arms extended. Her initial moment of inertia is

$$I_i = 3.2 \text{ kg·m}^2,$$

and her initial angular speed is

$$\omega_i = 1.5 \text{ rad/s}.$$

She pulls in her arms, reducing her moment of inertia to

$$I_f = 1.0 \text{ kg·m}^2.$$

No external torque acts on her. **Find her final angular speed** ω_f.

Solution.

Angular momentum is conserved:

$$L_i = L_f$$

$$I_i\omega_i = I_f\omega_f$$

Solve for the final angular speed:

$$\omega_f = \frac{I_i}{I_f}\,\omega_i$$

Substitute:

$$\omega_f = \frac{3.2}{1.0}(1.5) = 4.8 \text{ rad/s}$$

Answer:

$$\omega_f = 4.8 \text{ rad/s}$$