Midterm #1 - P3A - Version



Prof. Laszlo Bardoczi, October 19, 2025

- The exam is 40 minutes long and contains 4 calculation problems.
- You must solve 3 problems of your choice; 1 problem will not be graded.
- Each problem is worth up to 3 points (parts a, b & c): 0.5 point for a correct numerical value (to 2 significant figures); 0.5 point for correct SI units.

 Partial credit is awarded based solely on the boxed answers.
- Write final answers clearly in the designated boxes. Leave the boxes empty for the problem you do not wish to be graded. If all boxes are filled, Problem #4 will not be graded.
- A detachable formula sheet is provided at the end of the exam.
- At the end of the exam, submit only the front page and present your UCI ID for verification and sign the sign-in sheet the TAs will provide.
- Direct all grading questions to your TA.
- Cheating will result in an automatic failing grade for the course.

1. (a)	1. (b)	1. (c)
2. (a)	2. (b)	2. (c)
3. (a)	3. (b)	3. (c)
4. (a)	4. (b)	4. (c)

Problem 1

A spacecraft accelerating under constant thrust in deep space has its velocity described by

$$v(t) = 2.2t^3 - 5.5t^2 + 3.5t + 4.5,$$

where v is measured in m/s and t in seconds.

- (a) What is the velocity of the spacecraft at $t = 3.5 \,\mathrm{s}$?
- (b) How far does the spacecraft travel from t = 0 to t = 3.5 s?
- (c) What is the acceleration of the spacecraft at $t = 3.5 \,\mathrm{s}$?

Solution

(a)
$$v(3.5) = 2.2(3.5)^3 - 5.5(3.5)^2 + 3.5(3.5) + 4.5 = 94.3 - 67.4 + 12.3 + 4.5 = 44 \text{ m/s}.$$

(b)
$$x(t) = \int v(t) dt = \int (2.2t^3 - 5.5t^2 + 3.5t + 4.5) dt = 0.55t^4 - 1.83t^3 + 1.75t^2 + 4.5t.$$
 With $x(0) = 0$,
$$x(3.5) = 0.55(3.5)^4 - 1.83(3.5)^3 + 1.75(3.5)^2 + 4.5(3.5) = 82.5 - 78.4 + 21.4 + 15.8 \approx 41 \text{ m}.$$

(c)
$$a(t) = \frac{\mathrm{d}v}{\mathrm{d}t} = 6.6t^2 - 11t + 3.5.$$
 At $t = 3.5$,
$$a(3.5) = 6.6(3.5)^2 - 11(3.5) + 3.5 = 80.9 - 38.5 + 3.5 = 46 \,\mathrm{m/s}^2.$$

Such an acceleration corresponds to about 4.7 g, consistent with the thrust of a small launch vehicle.

Problem 2

On a space station biolab, a biologist studies how oxygen molecules diffuse through a spherical droplet of liquid containing single-celled algae. Assume: (1) step size $0.35 \,\mathrm{nm}$, (2) step time $8.0 \times 10^{-9} \,\mathrm{s}$, (3) motion in 3 D, (4) distance to chloroplast $1.2 \,\mu\mathrm{m}$.

- (a) Estimate the number of steps required for an oxygen molecule to reach the chloroplast.
- (b) For a second scenario, when diffusion takes 6.0×10^6 steps, calculate the total diffusion time (same step size, step time and dimensionality).
- (c) If the same process had to occur across the full droplet $(r_{\rm rms} = 1.6 \, \rm mm)$, how long would that take? For this question, give the answer in days.

Solution

$$r_{\rm rms} = d\sqrt{mn} \quad \Rightarrow \quad n = \frac{r_{\rm rms}^2}{md^2}.$$
(a)
$$n = \frac{(1.2 \times 10^{-6})^2}{3(0.35 \times 10^{-9})^2} = \frac{1.44 \times 10^{-12}}{3.68 \times 10^{-19}} = 3.9 \times 10^6.$$
(b)
$$t = n\tau = (6.0 \times 10^6)(8.0 \times 10^{-9}) = 4.8 \times 10^{-2} \,\mathrm{s} = 0.048 \,\mathrm{s}.$$
(c) For $r_{\rm rms} = 1.6 \times 10^{-3} \,\mathrm{m},$

$$n = \frac{(1.6 \times 10^{-3})^2}{3(0.35 \times 10^{-9})^2} = \frac{2.56 \times 10^{-6}}{3.68 \times 10^{-19}} = 7.0 \times 10^{12}.$$

$$t = n\tau = (7.0 \times 10^{12})(8.0 \times 10^{-9}) = 5.6 \times 10^4 \,\mathrm{s} = 0.65 \,\mathrm{days}.$$

Problem 3

An astronaut is playing golf on the Moon $(g = 1.70 \,\mathrm{m\,s^{-2}})$.

- (a) In the first hit, the horizontal velocity of the ball is $11.5\,\mathrm{m\,s^{-1}}$ and the vertical velocity is $7.5\,\mathrm{m\,s^{-1}}$ initially. What is the launch angle?
- (b) In the second hit, the vertical velocity is $7.5\,\mathrm{m\,s^{-1}}$ initially. How long until the ball lands?
- (c) In the third hit, the ball travels 110 m horizontally and reaches 18 m high. What was the launch speed?

Solution

(a)
$$\theta = \arctan\left(\frac{v_y}{v_x}\right) = \arctan\left(\frac{7.5}{11.5}\right) = 33^{\circ}.$$

(b)
$$T = \frac{2v_y}{g} = \frac{2(7.5)}{1.70} = 8.8 \,\mathrm{s}.$$

(c)
$$v_y = \sqrt{2gh} = \sqrt{2(1.70)(18)} = 7.8 \,\text{m/s}.$$

$$t_{\text{up}} = \frac{v_y}{g} = \frac{7.8}{1.70} = 4.6 \,\text{s}, \quad T = 9.2 \,\text{s}.$$

$$v_x = \frac{R}{T} = \frac{110}{9.2} = 12 \,\text{m/s}.$$

$$v_0 = \sqrt{v_x^2 + v_y^2} = \sqrt{12^2 + 7.8^2} = 14 \,\text{m/s}.$$

Problem 4: Proportional Reasoning — Fat Cells in the Human Body

The human body stores fat inside millions of microscopic spherical cells called *adipocytes*. Each adipocyte acts as a droplet of lipid, storing energy in the form of fat. (Help: $1 \text{ m}^3 = 1000 \text{ L}$)

- (a) Suppose an individual has a total of 4.5×10^{11} identical spherical fat cells. Together, these cells contain a total volume of stored fat equal to $8.0\,\mathrm{L}$. Assuming all the fat is contained inside the spherical cells, what is the diameter of one fat cell? Give your answer in $\mu\mathrm{m}$.
- (b) Now assume each fat cell has a radius of $r = 45 \,\mu\text{m}$ (micrometers). If there are 4.5×10^{11} such spherical cells in the body, what is the total surface area of all fat cells combined? Give your answer in square meters.
- (c) Measurements show that the combined surface area of all fat cells in a person is about $280 \,\mathrm{m}^2$. Assuming there are 4.5×10^{11} spherical fat cells that together make up this total area, what is the diameter of one cell? Give your answer in $\mu\mathrm{m}$.

Solution

(c)

(a)
$$V_{\text{tot}} = 8.0 \,\text{L} = 8.0 \times 10^{-3} \,\text{m}^3. \quad V_{\text{one}} = \frac{8.0 \times 10^{-3}}{4.5 \times 10^{11}} = 1.8 \times 10^{-14} \,\text{m}^3.$$

$$r = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3(1.8 \times 10^{-14})}{4\pi}\right)^{1/3} = 1.6 \times 10^{-5} \,\text{m}.$$

$$d = 2r = 3.2 \times 10^{-5} \,\text{m} = 32 \,\mu\text{m}.$$

(b)
$$A = 4\pi r^2 = 4\pi (45 \times 10^{-6})^2 = 2.5 \times 10^{-8} \,\mathrm{m}^2.$$

$$A_{\text{tot}} = (4.5 \times 10^{11})(2.5 \times 10^{-8}) = 1.1 \times 10^4 \,\mathrm{m}^2.$$

$$A_{\text{one}} = \frac{280}{4.5 \times 10^{11}} = 6.2 \times 10^{-10} \,\text{m}^2. \quad r = \sqrt{\frac{A}{4\pi}} = \sqrt{\frac{6.2 \times 10^{-10}}{4\pi}} = 7.0 \times 10^{-6} \,\text{m}.$$

$$d = 1.4 \times 10^{-5} \, \mathrm{m} = 14 \, \mu \mathrm{m}.$$