

P3A Midterm #1 Practice #2

Prof. L. Bardoczi

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Problem 1: Calcium Diffusion in a Neuron

When a synapse is activated, Ca^{2+} ions enter a neuron and then diffuse through the cytoplasm.

Assume: Step size in cytoplasm: $d = 0.30 \text{ nm}$. Step (collision) time: $\tau = 1.0 \times 10^{-8} \text{ s}$. Motion is 3D ($m = 3$).

- (a) How many steps n are needed for the RMS displacement to reach $r_{\text{rms}} = 2.0 \mu\text{m}$ (membrane $\rightarrow$ nearby target)?
- (b) How long does that diffusion take?
- (c) If instead the Ca^{2+} ion needed to reach a target 2.0 mm away (small tissue-scale path along a dendrite), how long would it take? Express your answer in days.

Worked Solution

The key relation for a 3D random walk is

$$r_{\text{rms}} = d\sqrt{mn} \implies n = \frac{r_{\text{rms}}^2}{md^2}$$

and the total diffusion time is $t = n\tau$.

Given (SI units)

$$d = 3.0 \times 10^{-10} \text{ m}, \quad \tau = 1.0 \times 10^{-8} \text{ s}, \quad m = 3$$

(a) Steps to reach $r_{\text{rms}} = 2.0 \mu\text{m}$

Convert distance:

$$r_{\text{rms}} = 2.0 \times 10^{-6} \text{ m}$$

$$n = \frac{(2.0 \times 10^{-6})^2}{3(3.0 \times 10^{-10})^2} = \frac{4.0 \times 10^{-12}}{2.7 \times 10^{-19}} \approx 1.48 \times 10^7$$

(b) Time for $2.0 \mu\text{m}$

$$t = n\tau = (1.48 \times 10^7)(1.0 \times 10^{-8} \text{ s}) = 1.48 \times 10^{-1} \text{ s} \approx 0.15 \text{ s}$$

Interpretation: That's fast on cellular timescales—consistent with rapid synaptic signaling.

(c) Time for 2.0 mm

Convert distance:

$$r_{\text{rms}} = 2.0 \times 10^{-3} \text{ m}$$

$$n = \frac{(2.0 \times 10^{-3})^2}{3(3.0 \times 10^{-10})^2} = \frac{4.0 \times 10^{-6}}{2.7 \times 10^{-19}} \approx 1.48 \times 10^{13}$$

$$t = n\tau = (1.48 \times 10^{13})(1.0 \times 10^{-8} \text{ s}) = 1.48 \times 10^5 \text{ s}$$

Convert to days:

$$\frac{1.48 \times 10^5 \text{ s}}{86400 \text{ s/day}} \approx 1.7 \text{ days}$$

Interpretation: Diffusion is excellent over microns but painfully slow over millimeters—exactly why cells and tissues rely on directed transport and circulation for longer distances.

Problem: Blood Cells & Surface Area (Proportional Reasoning)

Whole blood volume for an adult is about 5.0 L. Suppose the hematocrit (fraction of blood volume occupied by red blood cells) is 45 %. Model each red blood cell (RBC) as a sphere of diameter 8.0 μm .

1. Estimate the total number of RBCs in circulation.
2. What is the *total* surface area of all RBCs combined?
3. If you lined up all RBCs in single file, touching each other, how long would the line be?

Worked Solution

Given/Model:

$$\begin{aligned}V_{\text{blood}} &= 5.0 \text{ L} = 5.0 \times 10^{-3} \text{ m}^3, \\ \text{Hematocrit} &= 0.45 \Rightarrow V_{\text{RBC, total}} = 0.45 V_{\text{blood}} = 2.25 \times 10^{-3} \text{ m}^3, \\ d &= 8.0 \text{ } \mu\text{m} = 8.0 \times 10^{-6} \text{ m}, \quad r = 4.0 \times 10^{-6} \text{ m}, \\ V &= \frac{4}{3}\pi r^3, \quad A = 4\pi r^2.\end{aligned}$$

(a) Total number of RBCs

Volume of one RBC:

$$\begin{aligned}V_{\text{cell}} &= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4.0 \times 10^{-6})^3 \\ &= \frac{4}{3}\pi(64 \times 10^{-18}) \approx 2.68 \times 10^{-16} \text{ m}^3.\end{aligned}$$

Number of RBCs:

$$N = \frac{V_{\text{RBC, total}}}{V_{\text{cell}}} = \frac{2.25 \times 10^{-3}}{2.68 \times 10^{-16}} \approx 8.4 \times 10^{12} \text{ cells}.$$

(b) Total surface area

Area of one RBC:

$$A_{\text{cell}} = 4\pi r^2 = 4\pi(4.0 \times 10^{-6})^2 = 4\pi(16 \times 10^{-12}) \approx 2.01 \times 10^{-10} \text{ m}^2.$$

Total surface area:

$$A_{\text{total}} = N A_{\text{cell}} \approx (8.4 \times 10^{12})(2.01 \times 10^{-10}) \approx 1.7 \times 10^3 \text{ m}^2.$$

(This is roughly the area of several tennis courts, illustrating how massive surface area supports gas exchange—similar to the lung alveoli problem.)

(c) End-to-end length

If spheres just touch, each contributes one diameter d to the line length:

$$L = N d = (8.4 \times 10^{12})(8.0 \times 10^{-6} \text{ m}) \approx 6.7 \times 10^7 \text{ m} \approx 6.7 \times 10^4 \text{ km}.$$

Problem: The Courier's Pace

A medical courier jogs specimens between hospital buildings. She times 200 m in 36 s at a steady pace (assume constant speed; use ratios—no need to compute her speed explicitly).

- (a) How long would it take her to jog 800 m at the same pace?
- (b) How long would it take her to jog 1.00 km at the same pace?
- (c) She keeps this pace for 30 min. What speed must she run for the next 30 min so that her **average speed over the full hour** is 6.8 m s^{-1} ?

Worked Solution

For constant speed, time is directly proportional to distance:

$$\frac{t_2}{t_1} = \frac{d_2}{d_1}$$

Given

$$d_1 = 200 \text{ m}, \quad t_1 = 36 \text{ s}$$

(a) **Time for 800 m**

$$t_{800} = t_1 \cdot \frac{800}{200} = 36 \times 4 = \boxed{144 \text{ s}}$$

(b) **Time for 1.00 km**

$$t_{1000} = t_1 \cdot \frac{1000}{200} = 36 \times 5 = \boxed{180 \text{ s} = 3.00 \text{ min}}$$

(c) **Required second-half speed for average 6.8 m s^{-1}**

First 30 min (1800 s) at this pace:

$$v_1 = \frac{d_1}{t_1} = \frac{200}{36} = 5.56 \text{ m s}^{-1}$$

$$d_1 = v_1(1800) = (5.56)(1800) = 1.00 \times 10^4 \text{ m}$$

Total distance needed for a 1-hour average of 6.8 m s^{-1} :

$$d_{\text{target}} = (6.8)(3600) = 2.448 \times 10^4 \text{ m}$$

$$d_2 = d_{\text{target}} - d_1 = 24480 - 10000 = 1.448 \times 10^4 \text{ m}$$

Required speed in second 30 min:

$$v_2 = \frac{d_2}{1800} = \frac{14480}{1800} = \boxed{8.04 \text{ m s}^{-1}}$$

Interpretation: To raise her average speed from 5.56 m s^{-1} to 6.8 m s^{-1} , she must run significantly faster during the second half—just as a marathon runner must pick up the pace late in a race to meet an overall time goal.

Problem: Kangaroo-Rat Escape Jump (Projectile Motion)

A desert kangaroo rat makes a powerful escape jump. It travels a horizontal distance of $R = 4.8\text{ m}$ and reaches a maximum vertical height of $h = 1.2\text{ m}$ before landing back at the same ground level. Take $g = 9.8\text{ m s}^{-2}$.

- (a) What is the rat's **takeoff speed** v_0 ?
- (b) At what **launch angle** θ (above the horizontal) does it take off?
- (c) How long is the rat **in the air** (time of flight T)?

Worked Solution

We use standard projectile relations for same-height launch and landing. At maximum height, the vertical speed is zero and

$$h = \frac{v_y^2}{2g} \quad \text{so} \quad v_y = \sqrt{2gh}.$$

The total flight time is $T = \frac{2v_y}{g}$, the horizontal speed is $v_x = \frac{R}{T}$, and

$$v_0 = \sqrt{v_x^2 + v_y^2}, \quad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right).$$

1) Vertical launch component from max height

$$v_y = \sqrt{2gh} = \sqrt{2(9.8)(1.2)} = \sqrt{23.52} \approx 4.85\text{ m s}^{-1}.$$

2) Time to peak and total time of flight

$$t_{\uparrow} = \frac{v_y}{g} = \frac{4.85}{9.8} \approx 0.495\text{ s}, \quad T = 2t_{\uparrow} \approx 0.989\text{ s}.$$

3) Horizontal component from range

$$v_x = \frac{R}{T} = \frac{4.8}{0.989} \approx 4.85\text{ m s}^{-1}.$$

4) Takeoff speed and angle

$$v_0 = \sqrt{v_x^2 + v_y^2} = \sqrt{(4.85)^2 + (4.85)^2} = 4.85\sqrt{2} \approx 6.86\text{ m s}^{-1}.$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4.85}{4.85}\right) \approx 45^\circ.$$

Life Science Connection

Estimating v_0 and θ from trajectory data allows biologists to infer *muscle performance* and *power output* from high-speed videos of animal escapes. This is valuable for studying predator-prey dynamics and the biomechanics of locomotion.

Tracer Bolus in a Straight Microchannel

In a microfluidic experiment, a fluorescent tracer bolus moves along a straight microchannel. Its velocity along the x -axis is modeled (for $t \geq 0$) by

$$v(t) = -0.3t^2 + 1.8t + 0.6 \quad (\text{m s}^{-1}),$$

where t is in seconds. The tracer begins at the channel inlet at $t = 0$, so $x(0) = 0$.

- (a) Find the **position function** $x(t)$.
- (b) Find the **acceleration function** $a(t)$.
- (c) Compute the **displacement** and the **distance traveled** for $0 \leq t \leq 6$ s.
- (d) At what time is the tracer's **instantaneous velocity** maximal, and what is that value?
- (e) Find the **average velocity** on $0 \leq t \leq 6$ s.
- (f) Evaluate $x(4\text{ s})$.

Worked Solution

(a) Position from integrating velocity

$$x(t) = \int_0^t v(\tau) d\tau + x(0) = \int_0^t (-0.3\tau^2 + 1.8\tau + 0.6) d\tau.$$

Integrating term by term:

$$x(t) = \left(-0.1\tau^3 + 0.9\tau^2 + 0.6\tau \right) \Big|_{\tau=0}^{\tau=t} = -0.1t^3 + 0.9t^2 + 0.6t \quad (\text{m}).$$

(b) Acceleration from differentiating velocity

$$a(t) = \frac{dv}{dt} = -0.6t + 1.8 \quad (\text{m s}^{-2}).$$

(c) Displacement and distance on $[0, 6]$

Displacement:

$$x(6) = -0.1(6)^3 + 0.9(6)^2 + 0.6(6) = -21.6 + 32.4 + 3.6 = 14.4 \text{ m}.$$

Since $v(t) > 0$ for $0 \leq t \leq 6$, the distance traveled equals the displacement:

$\Delta x = 14.4 \text{ m}, \quad \text{Distance} = 14.4 \text{ m}.$
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(d) Time of maximum velocity

The parabola $v(t)$ has its maximum where $a(t) = 0$:

$$-0.6t + 1.8 = 0 \quad \Rightarrow \quad t = 3.0 \text{ s}.$$

Maximum velocity:

$$v(3) = -0.3(9) + 1.8(3) + 0.6 = -2.7 + 5.4 + 0.6 = \boxed{3.3 \text{ m s}^{-1}}.$$

(e) Average velocity on $[0, 6]$

$$\bar{v} = \frac{x(6) - x(0)}{6 - 0} = \frac{14.4}{6} = \boxed{2.4 \text{ m s}^{-1}}.$$

(f) Position at $t = 4$ s

$$x(4) = -0.1(64) + 0.9(16) + 0.6(4) = -6.4 + 14.4 + 2.4 = \boxed{10.4 \text{ m}}.$$

Final Results (SI units)

$$x(t) = -0.1 t^3 + 0.9 t^2 + 0.6 t,$$

$$a(t) = -0.6 t + 1.8,$$

$$\Delta x = 14.4 \text{ m}, \quad \text{Distance} = 14.4 \text{ m},$$

$$t_{\max v} = 3.0 \text{ s}, \quad v_{\max} = 3.3 \text{ m s}^{-1},$$

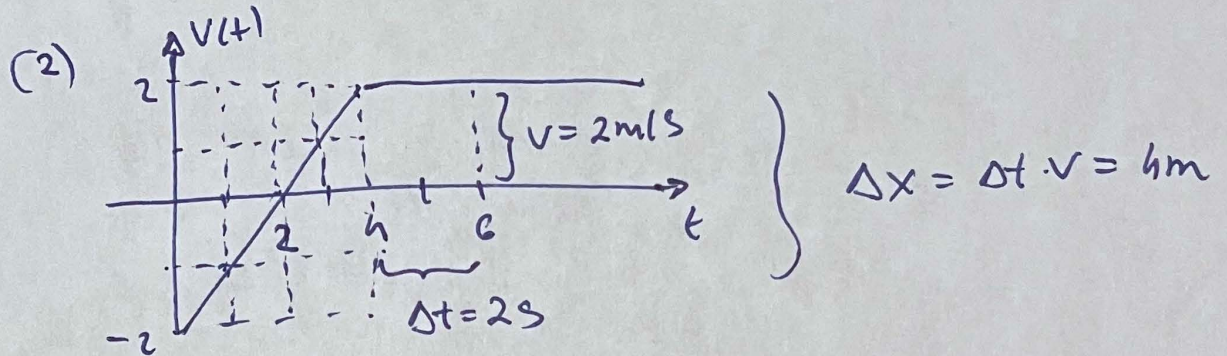
$$\bar{v} = 2.4 \text{ m s}^{-1}, \quad x(4) = 10.4 \text{ m}.$$

Life Science Connection

Polynomial velocity fits like this are often used to describe the motion of tracer particles in microfluidic or vascular flows. Integrating gives position (how far material has moved), while differentiating gives acceleration, linked to the driving pressure or pulsation of the flow—critical information in studying circulation or targeted drug delivery.

3.13 by train $V(t)$ graph

- given: (1) $X_0 = 2\text{m}$



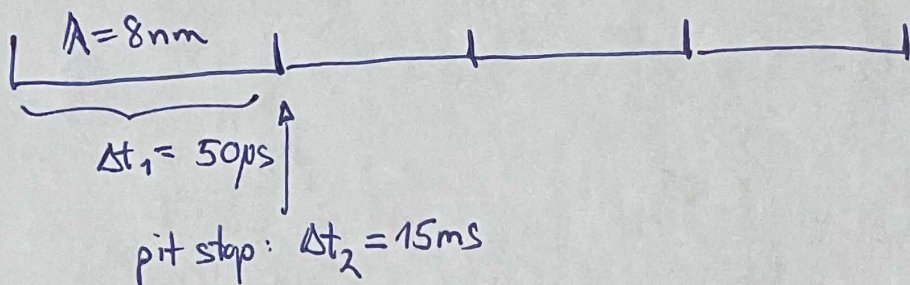
$$\Rightarrow X(6\text{sec}) = X_0 + \Delta X = 2\text{m} + 4\text{m} = 6\text{m}$$

3.27 Steelhead trout, $V_{y0} = 8\text{m/s}$, $y_{\text{max}} = ?$

$$y_{\text{max}} = \frac{V_{y0}^2}{2g} = \frac{8 \cdot 8}{2 \cdot 9.81} \approx 3.26\text{m}$$

Problem 2.10

Kinesin is a motor protein



• What is V_{av} ?

• def. : $V = \frac{\Delta x}{\Delta t}$

$$= \frac{\Delta x_1 + \Delta x_2}{\Delta t_1 + \Delta t_2} = \frac{8\text{nm}}{50\mu\text{s} + 15\text{ms}} = \frac{8 \cdot 10^{-9}\text{m}}{50 \cdot 10^{-6}\text{s} + 15 \cdot 10^{-3}\text{s}} =$$

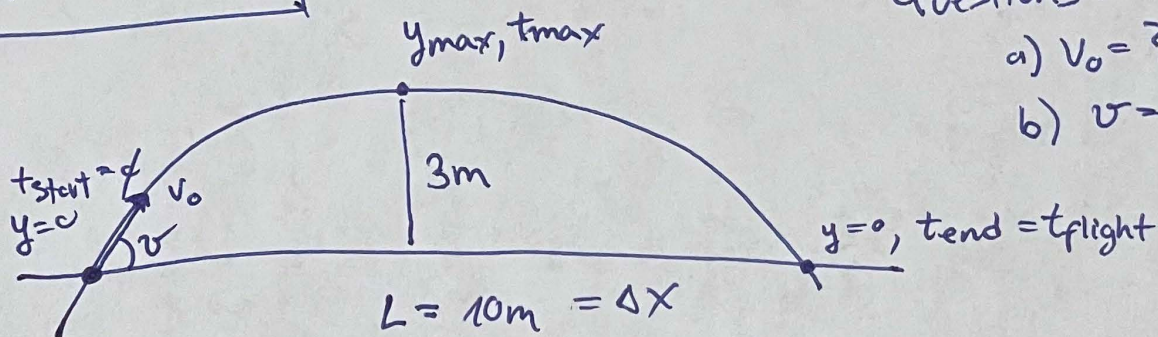
$15,05 \cdot 10^{-3}\text{s}$

$$= \frac{8}{15,05} \cdot \frac{10^{-9} \cdot 10^3}{10^{-6}} = 0,52 \cdot 10^{-6}\text{m/s}$$

$$= \underline{\underline{5,2 \cdot 10^{-7}\text{m/s}}}$$

3.35

Mountain lion



Questions:

a) $v_0 = ?$

b) $\theta = ?$

• from eq. sheet: a) $y_{\text{max}} = \frac{v_{y0}^2}{2g} \Rightarrow v_{y0} = \sqrt{y_{\text{max}} 2g} = \sqrt{3\text{m} \cdot 2 \cdot 9.81\text{m/s}^2}$
 $\approx 7.7\text{m/s}$

b) $t_{\text{flight}} = \frac{2v_{y0}}{g} = \frac{2 \cdot 7.7}{9.81} = \cancel{1.56\text{sec}} 1.56\text{sec}$

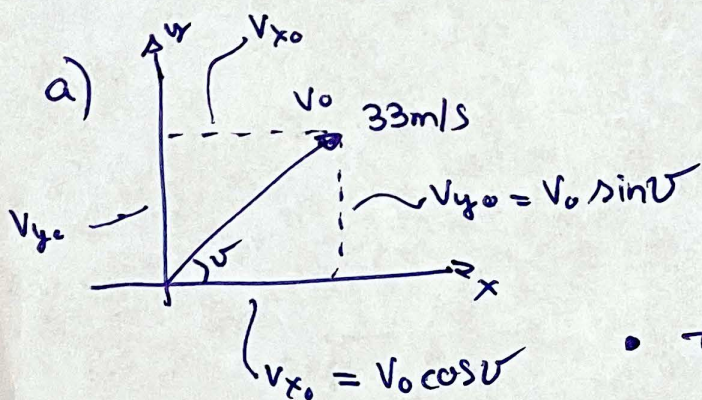
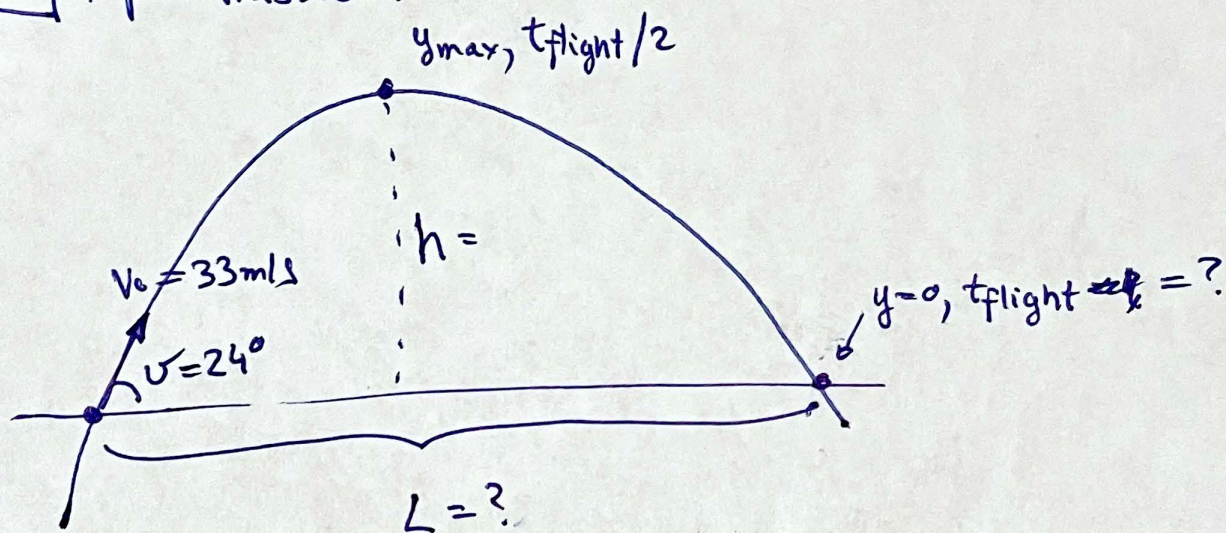
$\Delta x = v_{x0} t_{\text{flight}} \Rightarrow v_{x0} = \frac{\Delta x}{t_{\text{flight}}} = \frac{10\text{m}}{\cancel{1.56\text{sec}} 1.56\text{sec}} = \cancel{6.41\text{m/s}} 6.41\text{m/s}$

c) $v_0 = \sqrt{v_{x0}^2 + v_{y0}^2} = \sqrt{7.7^2 + 6.4^2} = 10\text{m/s}$

$\theta = \text{atan}\left(\frac{v_{y0}}{v_{x0}}\right) = 50.27^\circ$

3.38

Apollo mission



$$V_{x0} = 33 \cos 24 = 30.14 \text{ m/s}$$

$$V_{y0} = 33 \sin 24 = 13.4 \text{ m/s}$$

$$t_{\text{flight}} = \frac{2V_{y0}}{g} = \frac{2 \times 13.4}{9.81/6} = \frac{12 \times 13.4}{9.81} = 16.39 \text{ sec}$$

$$L = \Delta x = V_{x0} \cdot t_{\text{flight}} = 30.14 \text{ m/s} \cdot 16.39 \text{ sec} = 494 \text{ m}$$

$$L \text{ on Earth: } L_M = V_{x0} \cdot 2 \cdot V_{y0} / g_M \Rightarrow L_M \sim g_M^{-1}$$

$$L_E \sim g_E^{-1}$$

$$\Rightarrow L_E = \frac{1}{6} L_M = 82.3 \text{ m}$$

Derivation of random walk formula

(1-D):

- each step is $\Delta x_i = \pm d$ and $\langle \Delta x \rangle = 0$

- distance is $x = \sum_i \Delta x_i$

- $x_{\text{RMS}} = \sqrt{\langle x^2 \rangle}$

- $x^2 = \left(\sum_i \Delta x_i \right)^2 \Rightarrow \langle x^2 \rangle = \left\langle \left(\sum_i \Delta x_i \right)^2 \right\rangle = \left\langle \sum_i \Delta x_i^2 + 2 \underbrace{\sum_{i < j} \Delta x_i \Delta x_j}_{\phi} \right\rangle$

$\langle x^2 \rangle = \left\langle \sum_i \Delta x_i^2 \right\rangle = \left\langle d^2 \sum_i 1 \right\rangle = nd^2$

$\Delta x_i = \pm d$
 $\Delta x_i^2 = +d^2$

due to independence
of Δx_i and
 Δx_j
(correlation coeff)

$\Rightarrow x_{\text{RMS}} = \sqrt{\langle x^2 \rangle} = d\sqrt{n}$

m-dim : $\underline{x} = (x_1, x_2, \dots, x_m)$

$\langle \underline{x}^2 \rangle = \langle x_1^2 + x_2^2 + x_3^2 + \dots + x_m^2 \rangle = \underbrace{\langle x_1^2 \rangle}_{nd^2} + \underbrace{\langle x_2^2 \rangle}_{nd^2} \dots$

$= mnd^2$

$\Rightarrow x_{\text{RMS}} = d\sqrt{mn}$