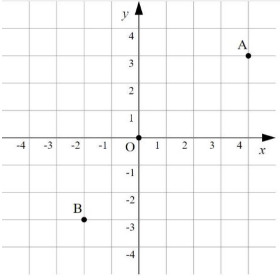
Physics 3A: Physics for the Life Sciences

Chapter 2: Describing Motion — Displacement, Velocity & Acceleration

Coordinate System

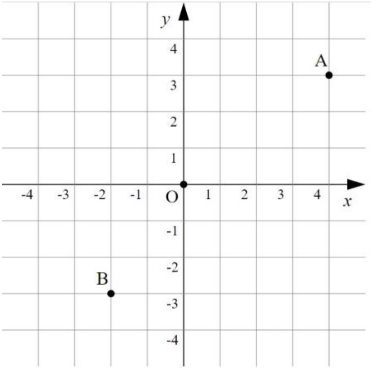
Coordinate System: A coordinate system enables to quantify the location, direction, and magnitude of physical quantities like position, velocity, force, and fields. It's an imaginary grid you center on the origin to measure position.

- **Reference Point (Origin):** The "starting point" to describe motion. (you choose) Example: the finish line of an F1 race, or the ground for a flying soccer ball.

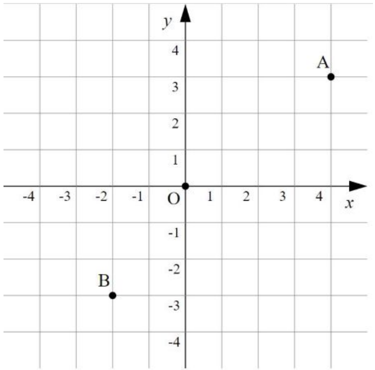


- **Horizontal axis** (x): left to right. (you choose)
- **Vertical axis** (y): bottom to top. (you choose)
- Coordinates are the numbers that show how far you are from the origin along each axis. (you choose their units)

Coordinate System



- What are the coordinates of points A & B?
- What will be the coordinates of points A & B if the origin is shifted one unit down and one unit to the left? (The points are fixed in the real physical world.)



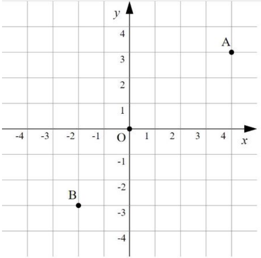
- What are the coordinates of points A & B?

  $A = (4, 3),\ B = (-2, -3)$

- What will be the coordinates of points A & B if the origin is shifted one unit down and one unit to the left? (The points are fixed in the real physical world.)

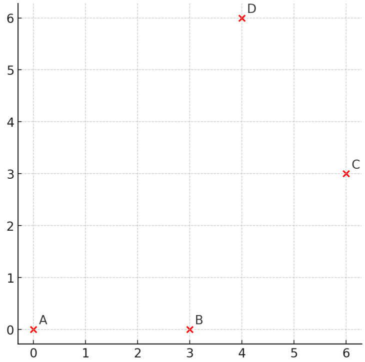
  $A = (5, 4),\ B = (-1, -2)$

- Motion is described by:
  - **Position** – where the object is (x, y).
  - **Displacement** – the change in position (how far and in what direction the object moved).
  - **Time** – when the object is there.
- Time as a coordinate:
  - Choose $t = 0$ as the start of observation.
  - Earlier instants $\rightarrow$ negative times.
- Example: start measuring time in an F1 race when the red lights go out.
- Negative values (for x, y, or t) are not "bad" – they simply locate events **relative to the chosen origin**.



Ant's Path: Displacement vs Position

An ant travels sequentially from point A to B, then to C, and finally to D. Determine the position vectors of all points and the displacement vectors corresponding to each segment of the journey.



Ant's Path: Displacement vs Position (See Details on YuJa)

- Points:

$$A(0,0), \quad B(3,0), \quad C(6,3), \quad D(4,6)$$

- Position vectors:

$$\vec{r}_A = (0,0), \quad \vec{r}_B = (3,0), \quad \vec{r}_C = (6,3), \quad \vec{r}_D = (4,6)$$

- Displacement vectors:

$$\vec{d}_{A,B} = (3,0), \quad \vec{d}_{B,C} = (3,3), \quad \vec{d}_{C,D} = (-2,3)$$

- Displacements add up to give position vectors:

$$\vec{r}_B = \vec{r}_A + \vec{d}_{A,B} = (0,0) + (3,0) = (3,0)$$

$$\vec{r}_C = \vec{r}_A + \vec{d}_{A,B} + \vec{d}_{B,C} = (0,0) + (3,0) + (3,3) = (6,3)$$

$$\vec{r}_D = \vec{r}_A + \vec{d}_{A,B} + \vec{d}_{B,C} + \vec{d}_{C,D} = (0,0) + (3,0) + (3,3) + (-2,3) = (4,6)$$

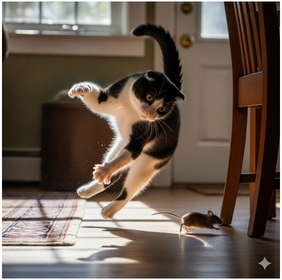
Scalars and Vectors

Scalar: A quantity described by a single number (with units).

- E.g.: heart rate, blood pressure, mass, temperature, time
- Can be positive, negative, or zero.

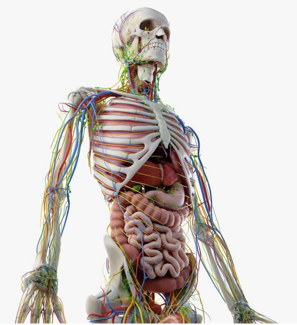
Vector: A quantity with both **magnitude** (size) and **direction**.

- Examples: displacement, velocity, force.
- Magnitude is always positive, direction tells "which way."
- Vectors are written with arrows: $\vec{a}$ or $\vec{A}$.
- They are represented by multiple numbers: each number describes the magnitude in the corresponding direction.
- E.g.: $v = (3, 4)$ m/s means 3 m/s in x and 4 m/s in y.



Examples on why vectors are needed:

- A nerve impulse travels 200 m/s — incomplete unless we know whether it goes **toward the brain** or **toward the muscle**.
- Multiple muscles pull the hip or shoulder at different angles; the **vector sum** determines the actual motion.



Examples on why vectors are needed:

- A nerve impulse travels 200 m/s — incomplete unless we know whether it goes **toward the brain** or **toward the muscle**.
- Multiple muscles pull the hip or shoulder at different angles; the **vector sum** determines the actual motion.
- Two cars moving at an intersection — same speed is not enough; their **directions** factor in whether they collide or pass safely.

- **Speed:**

$$v = \frac{\text{distance traveled}}{\Delta t}$$

  - Tells only "how fast."
  - Example: Driving 15 mi in 0.5 h $\rightarrow v = 30$ mph.
- **Velocity:**

$$\vec{v} = \frac{displacement}{\Delta t}$$

  - A vector — tells both "how fast" and "in which direction."
  - Example: Two ships move the same speed (20 mph) but in different directions:

$$\vec{v}_A = (0, 20 \text{ mph}) \text{ (going north)}, \quad \vec{v}_B = (20 \text{ mph}, 0) \text{ (going east)}$$

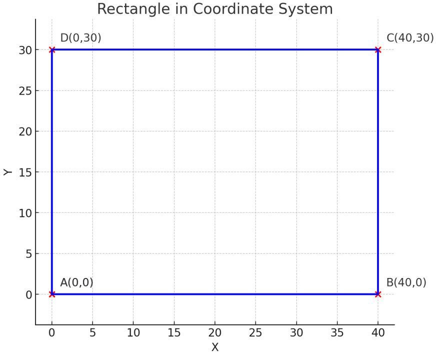
- **Key distinction:**
  - Speed = magnitude only.
  - Velocity = magnitude + direction. (how to remember? velocity starts with "v", like vector)
- **Key distinction (recap):**
  - Position = location of an object relative to the origin.
  - Displacement = change in position between two steps.

Ant on a Rectangle Path

An ant moves along the edges of a rectangle: 40 mm horizontally at 2 mm/s and 30 mm vertically at 1 mm/s. It reaches the rectangle's vertices at times $t_0, t_1, t_2, \ldots, t_4$.

Choose a coordinate system and for each vertex, determine:

a) The position vector
b) The displacement vector (from last vertex)
c) The distance traveled (since start)
d) The elapsed time (since start)
e) The average speed (since start)



Solution: Ant on a Rectangle Path (See Details on YuJa)

Setup: Rectangle 40 mm × 30 mm. Speeds: $v_x = 2\,\text{mm/s}$, $v_y = 1\,\text{mm/s}$.

(a) Position vectors

$$\vec{r}(t_0) = (0,0)\,\text{mm}$$
$$\vec{r}(t_1) = (40,0)\,\text{mm}$$
$$\vec{r}(t_2) = (40,30)\,\text{mm}$$
$$\vec{r}(t_3) = (0,30)\,\text{mm}$$
$$\vec{r}(t_4) = (0,0)\,\text{mm}$$

(b) Displacement (from last)

$$\Delta\vec{r}_{0\to1} = (40,0)\,\text{mm}$$
$$\Delta\vec{r}_{1\to2} = (0,30)\,\text{mm}$$
$$\Delta\vec{r}_{2\to3} = (-40,0)\,\text{mm}$$
$$\Delta\vec{r}_{3\to4} = (0,-30)\,\text{mm}$$

(c) Distance traveled (since start)

$$d(t_0) = 0\,\text{mm}$$
$$d(t_1) = 40\,\text{mm}$$
$$d(t_2) = 70\,\text{mm}$$
$$d(t_3) = 110\,\text{mm}$$
$$d(t_4) = 140\,\text{mm}$$

(d) Elapsed time (since start)

$$t_0 = 0\,\text{s}$$
$$t_1 = 20\,\text{s}$$
$$t_2 = 50\,\text{s}$$
$$t_3 = 70\,\text{s}$$
$$t_4 = 100\,\text{s}$$

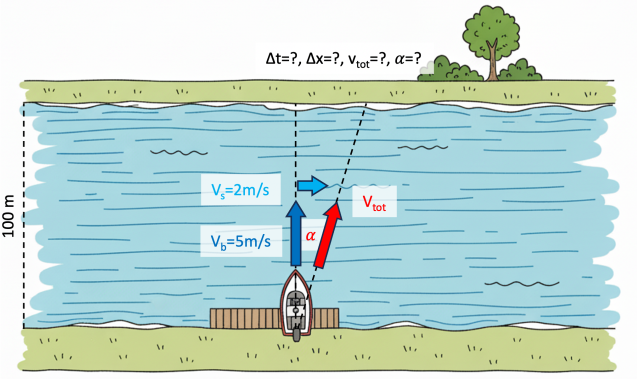
(e) Average speed (since start)

$$v_{\text{avg}}(t_0) = 0\,\text{mm/s}$$
$$v_{\text{avg}}(t_1) = \frac{40\,\text{mm}}{20\,\text{s}} = 2\,\text{mm/s}$$
$$v_{\text{avg}}(t_2) = \frac{70\,\text{mm}}{50\,\text{s}} = 1.4\,\text{mm/s}$$
$$v_{\text{avg}}(t_3) = \frac{110\,\text{mm}}{70\,\text{s}} \approx 1.57\,\text{mm/s}$$
$$v_{\text{avg}}(t_4) = \frac{140\,\text{mm}}{100\,\text{s}} = 1.4\,\text{mm/s}$$

Boat Crossing a River Without Correction

A boat must cross a river 100 m wide. Speed of boat in still water: 5 m/s. Speed of river current: 2 m/s. The boat is aimed **straight across** (perpendicular to the river bank), without correcting for the current.

a) Calculate the time it takes to cross the river.
b) Find how far downstream the boat drifts before reaching the opposite bank.
c) Determine the magnitude and direction of the boat's resultant velocity.



Solution: Boat Crossing a River Without Correction

Knowns: $v_b = 5\,\text{m/s}$ (boat speed), $v_r = 2\,\text{m/s}$ (river), width $= 100\,\text{m}$. The two motions in the two perpendicular direction (boat vs stream) are independent.

(a) Time to cross - Only perpendicular component matters. Distance & speed are given.

$$t = \frac{\text{width}}{v_b} = \frac{100}{5} = 20\,\text{s} \quad \longrightarrow \text{Boat lands in } 20\,\text{sec.}$$

(b) Downstream drift - Speed is given, we calculated the time already.

$$x = v_r \cdot t = 2 \cdot 20 = 40\,\text{m s} \quad \longrightarrow \text{Boat lands } 40\,\text{m downstream.}$$

(c) Resultant velocity (boat velocity + stream velocity)

$$v = \sqrt{v_b^2 + v_r^2} = \sqrt{5^2 + 2^2} = \sqrt{29} \approx 5.39\,\text{m/s}$$

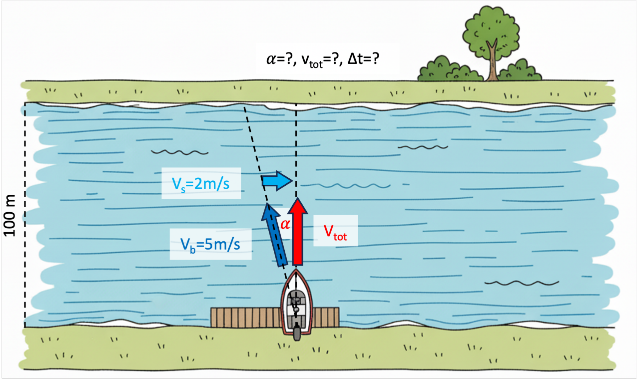
$$\theta = \arctan\left(\tfrac{2}{5}\right) \approx 21.8^\circ$$

Drift angle: 21.8° downstream.

Boat Crossing a River With Correction

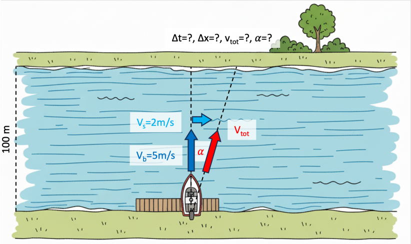
A boat must cross a river 100 m wide. Speed of boat in still water: 5 m/s. Speed of river current: 2 m/s. Goal: The boat must land **exactly across** from the starting dock (no drifting downstream).

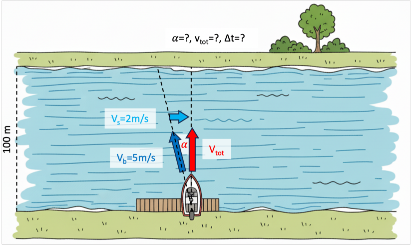
a) Find the angle at which the boat must head relative to the perpendicular (i.e., "aim upstream").
b) Calculate the effective velocity of the boat across the river.
c) Determine the time it takes to cross.



Compare the two cases

(Left) Uncompensated vs compensated (right)





Knowns: $v_b = 5\,\text{m/s}$ (boat speed), $v_r = 2\,\text{m/s}$ (river), width $= 100\,\text{m}$.

(a) Angle to aim upstream - We want the upstream boat velocity to be equal the river's downstream velocity so they perfectly cancel. That way, the boat will go straight through.

$$\sin\theta = \frac{v_r}{v_b} = \frac{2}{5} = 0.4$$

$$\theta = \arcsin(0.4) \approx 23.6^\circ$$

Boat must aim 23.6° upstream.

(b) Effective cross-river velocity

$$v_\perp = v_b \cos\theta = 5 \cdot \cos(23.6^\circ) \approx 4.58\,\text{m/s}$$

This is the speed across the river.

(c) Time to cross

$$t = \frac{\text{width}}{v_\perp} = \frac{100}{4.58} \approx 21.8\,\text{s}$$

So the boat reaches the opposite dock in ≈ 22 seconds.

Tricky Problem: Average Speed

A train travels from city A to city B:

- Distance A $\rightarrow$ B: 100 miles
- Speed A $\rightarrow$ B: 40 mph

Question: What must be the train's speed on the return trip (B $\rightarrow$ A) so that the **average speed for the whole journey** is 100 mph?

Solution: Average Speed Trap

Step 1: Average speed definition

$$\boxed{v_{\text{avg}} = \frac{\text{total distance}}{\text{total time} = \frac{\Delta x}{\Delta t}}}$$

Step 2: Required total time for 200 miles at 100 mph

$$t_{\text{req}} = \frac{200}{100} = 2\,\text{h}$$

Step 3: Time already used on first leg

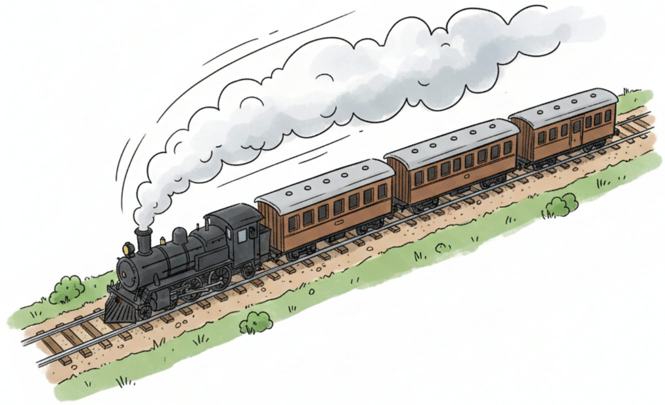
$$t_{AB} = \frac{100}{40} = 2.5\,\text{h}$$

Result: Already longer than t_{req}.

$\Rightarrow$ Impossible to achieve 100 mph average.

Another tricky one: What is the speed of a train's smoke?

Train goes at 50 mph along the track, wind blows at 20 mph perpendicular to the track. What will be the speed of the smoke?



An electric car drives from **San Diego** to **Salt Lake City** — a distance of **730 miles**. The car is initially fully charged.

- The car can travel **230 miles per charge** at an average speed of **65 mph**.
- A charging stop takes **40 minutes**. The car will fully recharge each time. Assume charging stations are available when needed.

Question: What is the car's **average velocity** for the entire trip?

Solution: Average Velocity on Road Trip

The average velocity is total distance divided by total time. The total distance is given: 730 miles. The total time is driving time + charging time.
The driving time is simply:

$$t_{\text{drive}} = \frac{\text{total distance}}{\text{drive speed}} = \frac{730 \text{ mi}}{65 \text{ mph}} = 11.23 \text{ h}$$

There will be three stops for charging (at 230 miles, 460 miles and 690 miles).
The time spend on these three stops is :

$$t_{\text{charge}} = 3 \times 0.67 = 2.01 \text{ h}$$

$$t_{\text{total}} = 11.23 + 2.01 = 13.24 \text{ h}$$

$$v_{\text{avg}} = \frac{\text{total distance}}{\text{total time}} = \frac{730 \text{ mi}}{13.24 \text{ h}} = \boxed{55.1 \text{ mph}}$$

Interpretation:

- Average velocity < 65 mph because charging stops increase total time.
- Even when the car isn't moving, time still counts!

Concept Check

- Average velocity depends on the **total displacement** and **total time**, not on the average of speeds.
- Stops (charging) increase total time $\Rightarrow$ reduce average velocity.
- Same reasoning applies in biology:
  - Blood flow averaged over time (rest + activity).
  - Drug diffusion or metabolic rate including rest intervals.

"Average" in physics always means total change divided by total time.

Introducing Acceleration

Why a new concept?

- So far: we described motion using **position**, **displacement**, **speed**, and **velocity**.
- But velocity itself can change with time — both in magnitude (speeding up / slowing down) and direction (turning).

Definition:

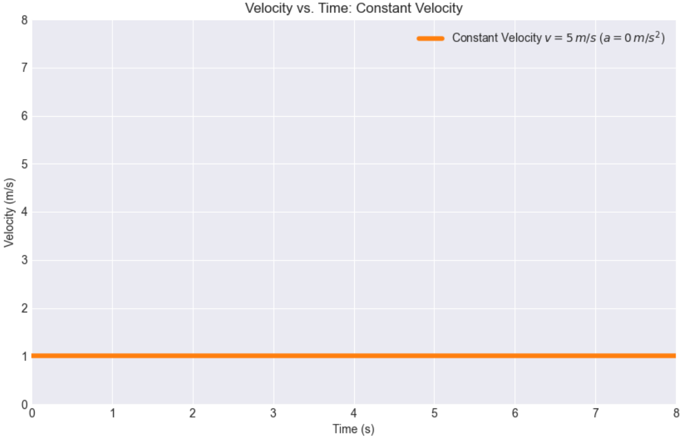
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

- $\vec{a}$ is the **acceleration vector**.
- Tells us how fast the velocity changes with time.
- Units: $\mathrm{m/s^2}$.

We will see: Acceleration connects *force* and *motion*.

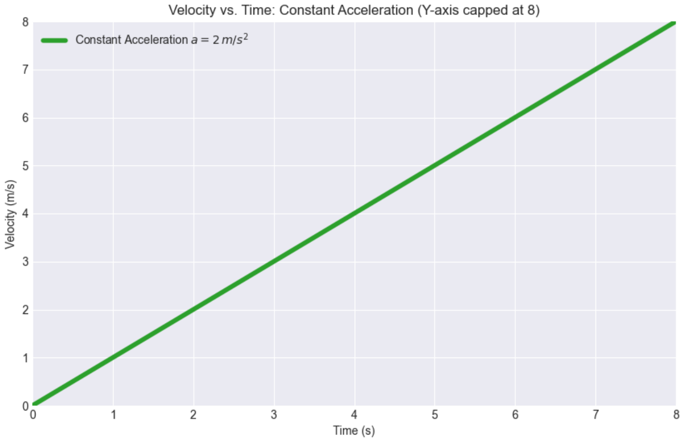
Distance with Constant Velocity Throughout

By definition, $v = \frac{\Delta x}{\Delta t} \longrightarrow \Delta x = v \Delta t$.
In words: the distance is equal to the area under the velocity graph.

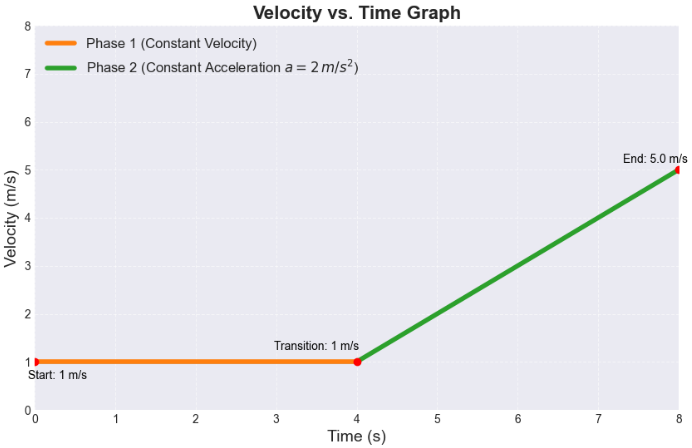


Distance with Constant Acceleration Throughout

By definition, $a = \frac{\Delta v}{\Delta t} \longrightarrow \Delta v = a\Delta t$.
The area under the velocity graph is $\Delta x = \frac{\Delta v \Delta t}{2}$ (think of a triangle).
This can be expressed using the acceleration as $\Delta x = \frac{1}{2}a\Delta t^2$.



If acceleration changes over time, brake the velocity graph into segments and add up the areas:
First part (orange): $\Delta x = v_1 \Delta t_1$ (time goes from 0 s to 4 s)
Second part(green) $\Delta x = v_1 \Delta t_2 + \frac{1}{2} a \Delta t_2^2$ (time goes from 4 s to 8 s)



Distance with Initial Velocity and Constant Acceleration

Situation: An object starts with an initial velocity v_i and accelerates at a constant rate a for a time Δt.

Formula for distance traveled:

$$s = v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$

Explanation:

- The first term $v_i \Delta t$ is the distance it would cover *without acceleration* (moving at constant speed v_i).
- The second term $\frac{1}{2} a (\Delta t)^2$ is the *extra distance* due to speeding up (or slowing down).

Units: Distance s is in meters (m), if v_i is in m/s, a in m/s^2, and t in seconds (s).

Example: Calculating Acceleration

A car speeds up from rest to 20 m/s in 5 s.
(a) What is the acceleration?
(b) What is the travelled distance?

Example: Calculating Acceleration and Distance

A car speeds up from rest to 20 m/s in 5 s.

(a) Acceleration

$$a = \frac{v_f - v_i}{\Delta t} = \frac{20 - 0}{5} = 4\,\text{m/s}^2$$

(b) Distance traveled

$$s = v_i \Delta t + \tfrac{1}{2} a (\Delta t)^2$$

$$s = 0 \cdot 5 + \tfrac{1}{2}(4)(5^2) = 50\,\text{m}$$

Interpretation: The car travels 50 m while accelerating to 20 m/s.