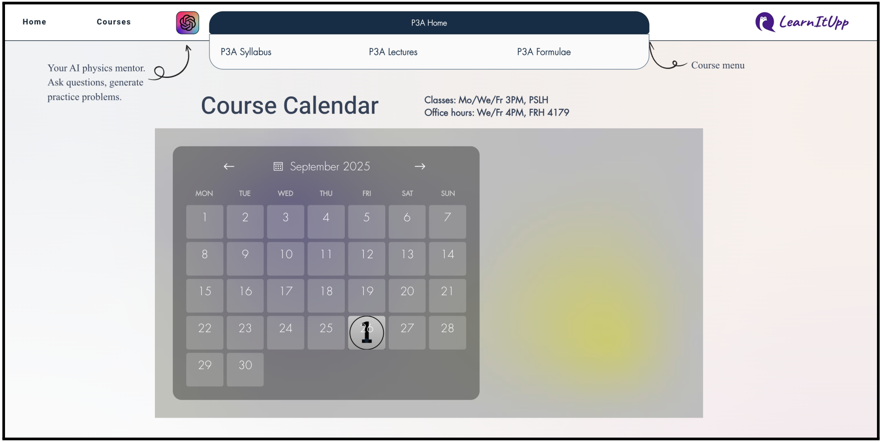
Physics 3A: Physics for the Life Sciences

Math Bootcamp

A free resource I created to support UCI students' learning. It provides the course calendar, syllabus, lecture notes, a custom ChatGPT trained on course content, and a formula sheet.



Math Review Topics

1. Algebra Refresher
   - Rearranging equations (solve for x, v, etc.)
   - Fractions, ratios, proportions (e.g. concentration of solutions)
   - Exponents and scientific notation (e.g. 10^{-6} M concentrations)
2. Functions and Graphs
   - Linear functions (slope/intercept $\rightarrow$ velocity vs. time graphs)
   - Quadratic/parabolic shape (projectile motion)
   - Exponential & logarithmic functions (radioactive decay, pH)
3. Trigonometry Basics
   - Right triangle, definitions of sin, cos, tan (using them to break vectors into components)
   - Example: force of gravity along an inclined plane $\rightarrow \sin\theta,\ \cos\theta$
4. Geometry
   - Pythagorean theorem (for magnitudes of vectors)
   - Circle basics: circumference, area, angles in radians (needed for oscillations & waves)
5. Vectors
   - Adding/subtracting vectors (graphical + component form)
   - Dot product in a simple way (projection, work $= F \cdot d$)
   - Example: displacement of a bacterium swimming in 2D
6. Basic Calculus "Lite" (Conceptual, Not Formal)
   - Derivative as slope/rate of change (velocity = slope of $x(t)$)
   - Integral as area/accumulation (distance traveled = area under $v(t)$)
   - Conceptual level with graphs instead of formal notation
7. Units & Dimensional Analysis
   - Converting units (mm $\rightarrow$ m, g $\rightarrow$ kg, etc.)
   - Checking equations for consistency (example: energy has units of Joules $= \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2$)
8. Statistics / Proportional Reasoning
   - Averages, uncertainty, orders of magnitude
   - Ratios (dose per body mass, scaling laws)

Algebra Essentials

- **Rearranging equations:** Solve $F = ma$ for m:

$$m = \frac{F}{a}$$

Click here to watch a related video:
Key Idea: Keep track of the unknown — move everything else to the other side.

- **Ratios & proportions:** If a car travels 60 km in 2 h, then in 5 h it travels:

$$\frac{60}{2} = \frac{x}{5} \quad \Rightarrow \quad x = 150\ \mathrm{km}$$

- **Exponents & scientific notation:**

$$6 \times 10^{-6} = 0.000006$$

$$x^a \cdot x^b = x^{a+b} \quad \text{e.g. } x^2 \cdot x^3 = x^5$$

$$(x^a)^b = x^{a \cdot b} \quad \text{e.g. } (y^2)^3 = y^6$$

$$\frac{x^a}{x^b} = x^{a-b} \quad \text{e.g. } \frac{z^5}{z^2} = z^3$$

Algebra Example: Dilution & Rearranging

- You have a concentrated solution at 2.0×10^{-3} mol/L.
  Say you need 50 mL at 5.0×10^{-5} mol/L.
- Use $C_i V_i = C_f V_f$, where C is concentration and V is volume (i: initial, f: final)
  Express the unknown on the left, move everything else to the right $\Rightarrow V_i = \dfrac{C_f V_f}{C_i}$.
- $V_i = \dfrac{(5.0 \times 10^{-5}\ \mathrm{mol/L})(50\ \mathrm{mL})}{2.0 \times 10^{-3}\ \mathrm{mol/L}} = 1.25\ \mathrm{mL}$ of stock.
- Fill to 50 mL with solvent (48.75 mL).

Result: Measure 1.25 mL stock, bring to 50 mL total.

Functions & Graphs

- Linear: $y = mx + b$ (slope m = rate; intercept b).
- Quadratic: $y = ax^2 + bx + c$ (parabolic shapes).

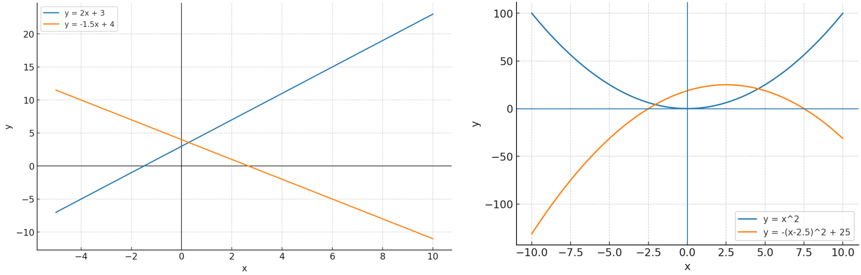


Figure: Linear and Quadratic Graphs

- Exponential: $y = Ae^{kt}$ (growth $k>0$, decay $k<0$); Log: $y = A - B \ln x$.
- Graph meaning in physics: slope $\to$ rate; area $\to$ accumulation.

Key Idea: Read slopes and areas to extract physical meaning.

Graph Example: Simple Projectile Motion

- Launch with horizontal velocity v_x and vertical velocity v_y.
- Equations of motion:

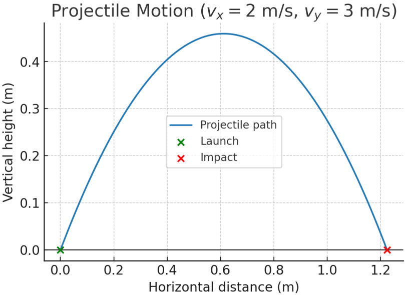
$$x(t) = v_x t, \quad y(t) = v_y t - \frac{1}{2} g t^2.$$

- Time of flight: solve $y(t) = 0$: $t_{\text{flight}} = \frac{2v_y}{g}$.
- Maximum height (at $t = v_y/g$): $y_{\max} = \frac{v_y^2}{2g}$.
- Horizontal range:

$$x_{\text{range}} = v_x t_{\text{flight}} = \frac{2v_x v_y}{g}.$$

- For $v_x = 2\ \mathrm{m/s}$, $v_y = 3\ \mathrm{m/s}$, $g = 9.8\ \mathrm{m/s^2}$:

$$t_{\text{flight}} \approx 0.61\ \mathrm{s}, \quad y_{\max} \approx 0.46\ \mathrm{m}, \quad x_{\text{range}} \approx 1.22\ \mathrm{m}.$$



Takeaway: Linear and quadratic functions are commonly used when calculating motion. Work symbolically, then plug in numbers at the end to minimize chances of errors.

Trigonometry Basics

- Right triangle: $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$.
- Components:

$$A_x = A\cos\theta,\ A_y = A\sin\theta$$

  (angle from x-axis).
- Radians: $2\pi\ \mathrm{rad} = 360^\circ$; use radians for oscillations and waves.

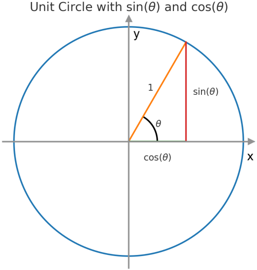


Figure: Sine and Cosine

- Object on a 30° incline. Weight mg splits into: $mg \sin\theta$ (down slope), $mg \cos\theta$ (into slope).
- If $m = 2.0$ kg, $g = 9.8$ m/s^2:
- Down-slope component: $F_{||} = mg \sin 30^\circ = 2.0 \cdot 9.8 \cdot 0.5 = 9.8$ N.
- Normal component: $F_\perp = mg \cos 30^\circ = 19.6 \cdot 0.866 \approx 17.0$ N.

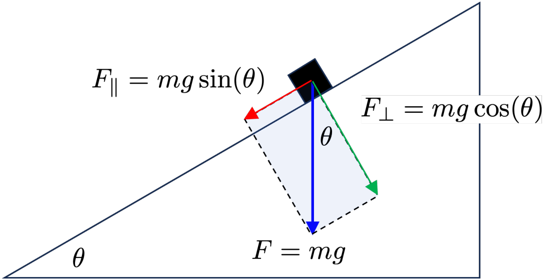


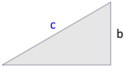
Figure: Sine and Cosine

Takeaway: Resolve vectors before applying Newton's laws.

Geometry Essentials

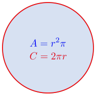
- Pythagorean theorem (a, b, c denote vector magnitudes or sides of a right triangle):

$$c^2 = a^2 + b^2$$



- Circles: circumference C, area A, arc length s with θ in radians:

$$C = 2\pi r, \quad A = \pi r^2, \quad s = r\theta$$

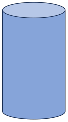


- Spheres & cylinders:

$$A_{\text{sphere}} = 4\pi r^2$$

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$V_{\text{cyl}} = \pi r^2 h$$



Key Idea: Learn a few shapes cold; they appear everywhere in physics (& bio).

Geometry Example: Cell Surface-To-Volume

- Model a bacterium as a sphere of radius $r = 1.0\ \mu\mathrm{m}$.
- Surface area: $A = 4\pi r^2 = 4\pi(1.0)^2 = 4\pi\ \mu\mathrm{m}^2$.
- Volume: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.0)^3 = \frac{4}{3}\pi\ \mu\mathrm{m}^3$.
- Ratio: $\frac{A}{V} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r} = 3\ \mu\mathrm{m}^{-1}$ (transport efficiency scales with A/V).

Takeaway: Smaller cells $\Rightarrow$ larger A/V $\Rightarrow$ can move nutrients, oxygen & waste in and out across its membrane faster.

Implications:

- **Cell size limit:** As cells get larger, A/V decreases $\Rightarrow$ slower diffusion per volume. Most cells remain microscopic.
- **Organ design:** Lungs (alveoli), intestines (villi), kidneys (nephrons) maximize surface area for efficient diffusion.
- **Large organisms:** Low A/V ratios overcome by circulatory systems that actively transport nutrients, gases, and wastes.

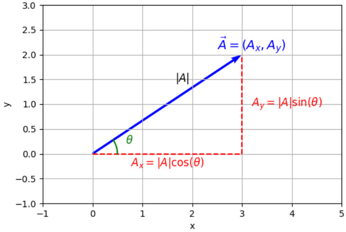
Takeaway: Surface-to-volume ratio explains why cells are small and why biological systems evolve structures to maximize exchange.

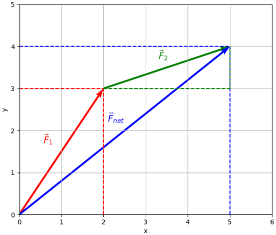
- **Climate adaptation:** A/V ratio also influences animal body shapes.
- **Cold climates (poles):** Animals tend to be larger and rounder. Smaller A/V ratio reduces heat loss (e.g. polar bears, penguins).
- **Warm climates (equator):** Animals tend to be slimmer with longer limbs/ears. Larger A/V ratio increases heat dissipation (e.g. gazelles, jackrabbits).
- This is known as **Allen's Rule** in biology.

Figure: Animal shape adeptation to climate

Takeaway: The same physics principle (A/V ratio) explains both cell limits and animal body shapes across climates.

- Vector $\vec{A} = (A_x, A_y) = \hat{\imath}A_x + \hat{\jmath}A_y$
- Magnitude of vector $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$
- Components of vector $A_x = |\vec{A}|\cos\theta, \quad A_y = |\vec{A}|\sin\theta$
- Angle of vector $\tan\theta = A_y/A_x$ or $\theta = \tan^{-1}(A_y/A_x)$.
- Addition: $\vec{A} + \vec{B} = (A_x + B_x, A_y + B_y)$.
- Dot product: $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = AB\cos\phi$ (projection; work).

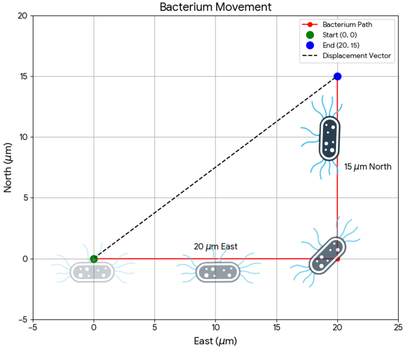




Key Idea: Always split vectors to components before combining vectors.

Vector Example: Net Displacement Of A Cell

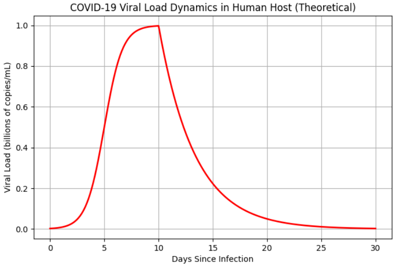
- A bacterium swims 20 $\mu\mathrm{m}$ east, then 15 $\mu\mathrm{m}$ north.
- How far did it get from the starting point, and in what direction?
- The two displacement vectors are $A_1 = (20, 0)\mu$ and $A_2 = (0, 15)\mu$
- $\vec{R} = (20, 15)\mu\mathrm{m}$; $|\vec{R}| = \sqrt{20^2 + 15^2} = 25\ \mu\mathrm{m}$.
- Direction: $\theta = \tan^{-1}\left(\frac{15}{20}\right) \approx 36.9^\circ$ north of east.

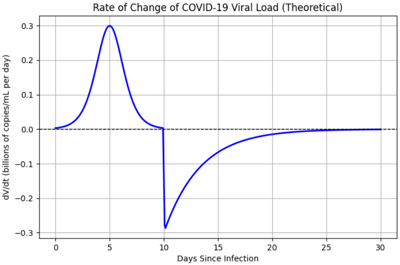


COVID-19 viral load in the body rises quickly as the virus multiplies, peaks around a week after infection, and then falls as the immune system clears it, much like a capacitor charging and discharging. COVID-19 viral load in the body rises quickly, peaks, and then falls as the immune system clears it.

- Change = slope: When the viral load curve $y(t)$ is steep, the virus is multiplying fast. When the curve flattens, growth has slowed or stopped. Similarly, on a distance–time graph, the slope is speed: a steeper line means faster motion.
- Total = area: The total number of viruses made is the area under the replication-rate curve. Likewise, the total distance traveled is the area under a velocity–time graph.

Key Idea: Slope tells how fast things change; area tells the total accumulated amount.



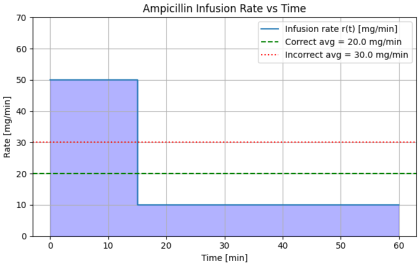


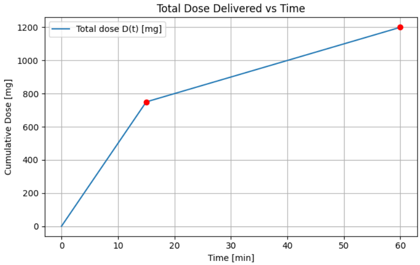
Calculus Example: Total Antibiotic Dose

- A patient receives **ampicillin** intravenously. The infusion rate is

$$r(t) = \begin{cases} 50\ \mathrm{mg/min}, & 0 \le t \le 15\ \mathrm{min}, \\ 10\ \mathrm{mg/min}, & 15 < t \le 60\ \mathrm{min}. \end{cases}$$

- Total dose $= (50)(15) + (10)(45) = 750 + 450 = 1200\ \mathrm{mg}$.
- Average infusion rate: $\dfrac{\text{total dose}}{\text{total time}} = \dfrac{1200\ \mathrm{mg}}{60\ \mathrm{min}} = 20\ \mathrm{mg/min}$.
- Watch out: the average is not the average of the two doses! (that's 30 mg/min)
  The average dose is the total dose per total time!





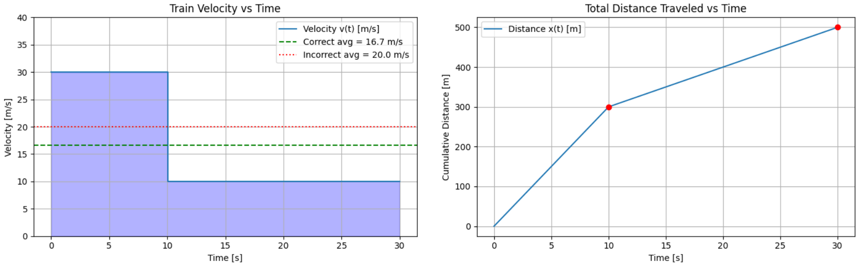
Takeaway: Integrating drug infusion rate gives the total IV dose delivered.

Let's apply this to physics next!

- A train travels with the following velocity:

$$v(t) = \begin{cases} 30 \text{ m/s}, & 0 \le t \le 10 \text{ s}, \\ 10 \text{ m/s}, & 10 < t \le 30 \text{ s}. \end{cases}$$

- Total distance $= (30)(10) + (10)(20) = 300 + 200 = 500$ m.
- Average speed: $\dfrac{\text{total distance}}{\text{total time}} = \dfrac{500 \text{ m}}{30 \text{ s}} \approx 16.7$ m/s.
- Watch out: the average is not the average of the two speeds! (that's 20 m/s) The average speed is the total distance per total time!



Takeaway: Integrating velocity gives the total distance traveled.
Same math as in biology — just a different context!

- **SI base units:** m (length), kg (mass), s (time), A (current), K (temperature), mol (amount), cd (luminous intensity).
- **Prefixes (powers of 10) — Essential to memorize:**

n	μ	m	c	d	k	M	G
10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^{-1}	10^{3}	10^{6}	10^{9}

The World Health Organization states that in the U.S., medication errors injure approximately 1.3 million people annually, with about one death occurring daily. Some of these are due to confusion over measurement units - like mixing up teaspoons vs. milliliters or pounds vs. kilograms.

- **Unit conversion:** set up as fractions that equal 1 (cancel units). E.g.:

$$15 \ \mathrm{s} \times \frac{6 \ \mathrm{m}}{1 \ \mathrm{s}} = 90 \ \mathrm{m}$$

- **Dimensional check:** both sides of an equation must have same units. E.g. for $s = vt$, LHS: $[s] = \mathrm{m}$, RHS: $[vt] = \frac{\mathrm{m}}{\mathrm{s}} \cdot \mathrm{s} = \mathrm{m}$.

Key Idea: Treat units like algebraic symbols; they multiply, divide, and cancel.

Examples: Units, Conversions, Dimensional Checks

Units:

- **Kinetic energy:** $K = \frac{1}{2}mv^2$ Units: $\mathrm{kg} \cdot (\mathrm{m/s})^2 = \mathrm{kg\,m^2/s^2} = \mathrm{J}$ (checks out).
- **Force:** $F = ma$ Units: $\mathrm{kg} \cdot \mathrm{m/s^2} = \mathrm{N}$ (newton).
- **Work = Force $\times$ Distance:** $W = Fd$ Units: $\mathrm{N} \cdot \mathrm{m} = \mathrm{J}$ (consistent with energy).

Conversion examples

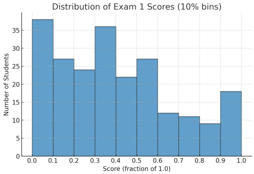
- **Mass:** $250\ \mathrm{mg} \times \dfrac{1\ \mathrm{g}}{1000\ \mathrm{mg}} \times \dfrac{1\ \mathrm{kg}}{1000\ \mathrm{g}} = 2.5 \times 10^{-4}\ \mathrm{kg}.$
- **Speed:** $72\ \mathrm{km/h} \times \dfrac{1000\ \mathrm{m}}{1\ \mathrm{km}} \times \dfrac{1\ \mathrm{h}}{3600\ \mathrm{s}} = 20\ \mathrm{m/s}.$

Dimensional sanity checks:

- **Spring energy:** $U = \frac{1}{2}kx^2$ Units: $(\mathrm{N/m}) \cdot \mathrm{m^2} = \mathrm{N} \cdot \mathrm{m} = \mathrm{J}.$
- **Momentum:** $p = mv$ Units: $\mathrm{kg} \cdot \mathrm{m/s}$ (no special name, but consistent with Newton's laws).

Takeaway: Units behave like algebra: they multiply, divide, and cancel — a built-in error check for every formula.

- **Statistics:** Average (mean) gives a typical value. Spread (range or standard deviation) shows variability. Example: class test scores between 0% and 100%, mean = 40%, SD = 27.



- **Proportionality:**
  - $y \propto x \rightarrow$ double x, y doubles (linear). Example: acceleration $\propto F$.
    Example: Tylenol dosage in children scales with body mass (15 $\mathrm{mg/kg}$ every 6 hours).

$$\text{25 lb child} \approx 11.4\ \mathrm{kg} \quad \Rightarrow \quad 15 \times 11.4 \approx 170\ \mathrm{mg}$$

$$\text{45 lb child} \approx 20.5\ \mathrm{kg} \quad \Rightarrow \quad 15 \times 20.5 \approx 310\ \mathrm{mg}$$

  - $y \propto x^2 \rightarrow$ double x, y quadruples (quadratic). Example: kinetic energy $\propto v^2$
  - $y \propto 1/x \rightarrow$ double x, y halves (inverse). Example: gravitational force $\propto 1/r^2$.
- **Orders of magnitude:** Compare scales by powers of 10. Example: a cell (10^{-5} m) vs. a human (1 m) $\rightarrow$ difference of 10^5 in size. A proton mass vs. a human mass differs by $\sim 10^{27}$.

Key Idea: Know how outputs scale when inputs change — and check if results make sense in size.