Physics 3A: Physics for the Life Sciences

Chapter 5: Interacting System

Laszlo Bardoczi

Net Force and Acceleration in One Dimension

Two horizontal forces act on a 2.0 kg object:

$$F_1 = 5\,\text{N (to the right)}, \quad F_2 = 3\,\text{N (to the left)}$$

Find the magnitude and direction of the acceleration.

Net Force:

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = (5 - 3)\,\text{N} = 2\,\text{N (to the right)}$$

Newton's Second Law:

$$a = \frac{F_{\text{net}}}{m} = \frac{2}{2.0} = 1.0\,\text{m/s}^2$$

Result:

$$\boxed{a = 1.0\,\text{m/s}^2 \text{ to the right}}$$

Net Force and Acceleration in Two Dimensions

A 2.0 kg object is pulled by two forces:

$$|F_1| = 5\,\mathrm{N}, \quad |F_2| = 3\,\mathrm{N}$$

F_1 is at 45° above the x-axis, F_2 is at 25° below the x-axis.

Find the acceleration vector.

Force components:

$$F_{1x} = F_1 \cos 45^\circ, \quad F_{1y} = F_1 \sin 45^\circ,$$
$$F_{2x} = F_2 \cos 25^\circ, \quad F_{2y} = -F_2 \sin 25^\circ.$$

Net force components:

$$F_{\text{net},x} = F_{1x} + F_{2x}, \quad F_{\text{net},y} = F_{1y} + F_{2y}.$$

Acceleration:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \left(\frac{F_{1x} + F_{2x}}{m}, \frac{F_{1y} + F_{2y}}{m} \right)$$

Magnitude:

$$a = \sqrt{a_x^2 + a_y^2}$$

Ball Kicked With a Force for a Short Time

A soccer ball of mass $m = 0.45\,\text{kg}$ is kicked with a powerful force of $F = 2000\,\text{N}$.

The contact time between the foot and ball is $\Delta t_k = 0.01\,\text{s}$.

The ball then flies freely for $\Delta t = 2.0\,\text{s}$.

Find (a) the launch speed v_0 and (b) the distance traveled.

(a) Acceleration and Launch Speed

$$a = \frac{F}{m} = \frac{2000}{0.45} = 4.44 \times 10^3\,\text{m/s}^2$$

$$v_0 = a\Delta t_k = (4.44 \times 10^3)(0.01) = 44.4\,\text{m/s}$$

(b) Distance Traveled

$$x = \tfrac{1}{2}a\Delta t_k^2 + v_0(\Delta t - \Delta t_k)$$

$$x = \tfrac{1}{2}(4.44 \times 10^3)(0.01)^2 + (44.4)(2.0 - 0.01)$$

$$x = 0.222 + 88.36 \approx \boxed{88.6\,\text{m}}$$

Inclined Plane and Condition for Sliding

A block rests on an incline at angle α.

What are the forces acting on it?

Under what condition will it begin to slide?

Forces on the block:

$$\begin{aligned} F_g &= mg, \\ F_N &= mg\cos\alpha, \\ F_{g,\parallel} &= mg\sin\alpha, \\ F_f &= \mu_s F_N = \mu_s mg\cos\alpha. \end{aligned}$$

Condition for equilibrium:

$$F_f = F_{g,\parallel} \quad \Rightarrow \quad \mu_s mg\cos\alpha = mg\sin\alpha$$

Solve for the critical angle:

$$\boxed{\mu_s = \tan\alpha}$$

Interpretation: The block begins to slide when the component of gravity along the incline equals the maximum static friction.

Equivalent Spring Constant: Parallel Combination

Two identical springs of constant k each are attached in parallel to the same mass.

Both springs compress by the same amount Δx.

Forces:

$$F_1 = -k\Delta x, \quad F_2 = -k\Delta x$$

Total force:

$$F = F_1 + F_2 = -2k\Delta x$$

Equivalent spring constant:

$$\boxed{K = 2k}$$

In general:

$$K = k_1 + k_2$$

Equivalent Spring Constant: Series Combination

Two springs with constants k_1 and k_2 are connected in series.

The same force F acts on both, but each stretches differently.

Force balance:

$$F = k_1 \Delta x_1 = k_2 \Delta x_2$$

$$\Delta x = \Delta x_1 + \Delta x_2$$

Total extension:

$$\frac{F}{K} = \frac{F}{k_1} + \frac{F}{k_2}$$

Equivalent spring constant:

$$\boxed{\frac{1}{K} = \frac{1}{k_1} + \frac{1}{k_2}}$$

Special case: if $k_1 = k_2 = k$, then $K = \frac{1}{2}k$

Condition for Lifting With Two Ropes

A box is pulled upward by two ropes under tension T at angle θ.

Each rope provides a vertical component $T \sin\theta$.

Vertical force balance:

$$2T \sin\theta > mg \quad \text{(required to lift the box)}$$

At equilibrium (just about to lift):

$$2T \sin\theta = mg$$

Horizontal components cancel:

$$T \cos\theta \text{ (left)} = T \cos\theta \text{ (right)}$$

Two blocks m_1 and m_2 rest on an incline of angle α.

The blocks are connected by two springs with constants k_1 and k_2.

Both blocks experience friction with coefficient μ.

The friction is too small to hold the blocks.

Find the equilibrium extensions Δx_1 and Δx_2.

For Block m_2:

$$k_2 \Delta x_2 = m_2 g(\sin\alpha - \mu\cos\alpha)$$

For Block m_1:

$$k_1 \Delta x_1 = m_1 g(\sin\alpha - \mu\cos\alpha) + k_2 \Delta x_2$$

Substitute for Δx_2:

$$\Delta x_1 = \frac{1}{k_1}\left[m_1 g(\sin\alpha - \mu\cos\alpha) + m_2 g(\sin\alpha - \mu\cos\alpha)\right]$$

Simplify:

$$\boxed{\Delta x_1 = \frac{(m_1 + m_2) g(\sin\alpha - \mu\cos\alpha)}{k_1}}$$

Interpretation: It's as if both masses were attached directly to the first spring — the second spring transmits the same effective load.

Two-Mass System on an Incline (With and Without Friction)

Two masses are connected by a light string over a frictionless pulley.
m_1 is on an incline of angle α, m_2 hangs vertically.
We seek the condition for equilibrium.

(a) Without Friction:

$$\Sigma F_{||} = 0 \quad \Rightarrow \quad m_2 g = m_1 g \sin\alpha$$

$$\boxed{\sin\alpha = m_2/m_1}$$

If $m_2 > m_1 \sin\alpha$: system moves downward (mass m_2 descends)
If $m_2 < m_1 \sin\alpha$: system moves upward (mass m_2 rises)

(b) With Friction:

At threshold of motion, $F_f = \mu_s F_N = \mu_s m_1 g \cos\alpha$

Case 1: m_2 tends to move down

$$m_2 g = m_1 g \sin\alpha + \mu_s m_1 g \cos\alpha \quad \Rightarrow \quad \mu_s = \frac{m_2/m_1 - \sin\alpha}{\cos\alpha}$$

Case 2: m_2 tends to move up

$$m_2 g = m_1 g \sin\alpha - \mu_s m_1 g \cos\alpha \quad \Rightarrow \quad \mu_s = \frac{\sin\alpha - m_2/m_1}{\cos\alpha}$$

Condition for equilibrium:

$$\boxed{\mu_s \geq \frac{\left|\frac{m_2}{m_1} - \sin\alpha\right|}{\cos\alpha}}$$

Block on a Slide in an Accelerating Elevator

A block of mass $m = 2.0\,\mathrm{kg}$ rests on a smooth incline ($\theta = 30^\circ$) inside an elevator.

The elevator accelerates **upward** with $a_{\mathrm{el}} = 2.0\,\mathrm{m/s^2}$.

Find the block's acceleration **along the incline** as seen by someone inside the elevator.

Step 1: Forces in the Elevator Frame (Non-Inertial):

$$\text{Real forces: } \begin{cases} W = mg \text{ (downward)} \\ N = \text{normal force (perpendicular to incline)} \end{cases}$$

$$\text{Fictitious force: } F_{\mathrm{fict}} = -ma_{\mathrm{el}} \text{ (downward if elevator accelerates up)}$$

Step 2: Effective Gravity in Elevator Frame:

$$g_{\mathrm{eff}} = g + a_{\mathrm{el}} = 9.8 + 2.0 = 11.8\,\mathrm{m/s^2}$$

Step 3: Acceleration Down the Incline:

$$a_{\mathrm{block}} = g_{\mathrm{eff}} \sin\theta = 11.8 \sin 30^\circ = 5.9\,\mathrm{m/s^2}$$

Compare: If the elevator were at rest, $a = g\sin\theta = 4.9\,\mathrm{m/s^2}$ — so the block slides faster because the elevator is accelerating upward.

Apparent Weight in an Accelerating Elevator

A 5.0 kg mass hangs from a spring scale in an elevator.

The elevator accelerates **downward** at $a = 3.0\,\mathrm{m/s^2}$.

What is the **apparent weight** shown by the scale?

Forces:

$$T - mg = ma \quad \Rightarrow \quad T = m(g - a)$$

Plug in:

$$T = 5.0(9.8 - 3.0) = 34\,\mathrm{N}$$

Interpretation: The apparent weight is smaller than the true weight (49 N) because the elevator accelerates downward — the scale reads less.

Spring Constant to Prevent the Block from Falling

A block of mass $m = 1.0\,\text{kg}$ is pressed against a vertical wall by a spring.

The spring is compressed by $\Delta x = 0.02\,\text{m}$.

The coefficient of static friction between the block and wall is $\mu = 0.2$.

What spring constant k ensures that the block does **not fall**?

Forces:

$$F_g = mg,$$
$$F_{\text{fr}} = \mu F_N,$$
$$F_N = F_{\text{sp}} = k\Delta x.$$

Condition for equilibrium:

$$F_{\text{fr}} = F_g \quad \Rightarrow \quad \mu k \Delta x = mg$$

Solve for the spring constant:

$$k = \frac{mg}{\mu\,\Delta x}$$

Substitute:

$$k = \frac{(1.0)(9.8)}{(0.2)(0.02)} = 2450\,\text{N/m}$$

Ball Falling With Drag

Drag force: $F_D = -cv$ (low Reynolds number regime)
Gravitational force: $F_g = mg$
Equation of motion: $\sum F = ma = mg - cv$

(a) Terminal Speed

$$mg = cv_t \quad \Rightarrow \quad v_t = \frac{mg}{c}$$

(b) Velocity as a Function of Time

$$a(t) = \frac{dv}{dt} = g - \frac{c}{m}v$$

$$\frac{dv}{g - \frac{c}{m}v} = dt$$

Integrate:

$$-\frac{m}{c}\int_0^v \frac{dv'}{v_t - v'} = \int_0^t dt'$$

$$-\frac{m}{c}\ln\left|1 - \frac{v}{v_t}\right| = t$$

Solve for $v(t)$:

$$v(t) = v_t\left(1 - e^{-\frac{c}{m}t}\right)$$

Interpretation:

At $t = 0$: $v(0) = 0$ (starts from rest)
As $t \to \infty$: $v(t) \to v_t$ (reaches terminal speed)

Vertical Spring Compression at Equilibrium

A block of mass m rests on a vertical spring.

What are the forces? What is the final compression Δx?

Forces:

$$F_g = mg, \quad F_s = -k\Delta x$$

At equilibrium: $\sum F = 0$

$$mg = k\Delta x$$

Solve for spring compression:

$$\boxed{\Delta x = \frac{mg}{k}}$$

Spring Compression on an Incline With Friction

A block of mass m rests on an incline of angle θ, attached to a spring.

The coefficient of friction is μ.

Find the equilibrium compression Δx.

Forces along the incline:

$$F_{\text{sp}} = -k\Delta x, \quad F_g = mg\sin\theta, \quad F_{\text{fr}} = \mu F_N = \mu mg\cos\theta$$

At equilibrium: $\sum F_{\parallel} = 0$

$$mg\sin\theta - \mu mg\cos\theta - k\Delta x = 0$$

Solve for compression:

$$\boxed{\Delta x = \frac{mg(\sin\theta - \mu\cos\theta)}{k}}$$

Harmonic Oscillation (Introduction)

A block of mass m attached to a spring (spring constant k) moves horizontally without friction.
The restoring force is given by Hooke's law:

$$F = -kx$$

Newton's Second Law:

$$F = ma = m\frac{d^2x}{dt^2} \quad \Rightarrow \quad -kx = m\frac{d^2x}{dt^2}$$

Equation of motion:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

Solution form:

$$x(t) = A\sin(\omega t) \quad \text{or} \quad x(t) = A\cos(\omega t)$$

where

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

Velocity and acceleration:

$$v(t) = \frac{dx}{dt} = A\omega\cos(\omega t), \quad a(t) = \frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t)$$

Key points:

Maximum displacement $\Delta x_{\text{max}} = A$ occurs at $v = 0$.
Maximum velocity $v_{\text{max}} = A\omega$ occurs at $x = 0$.