Name: **ID:**

Midterm #2 - P3A - Version B

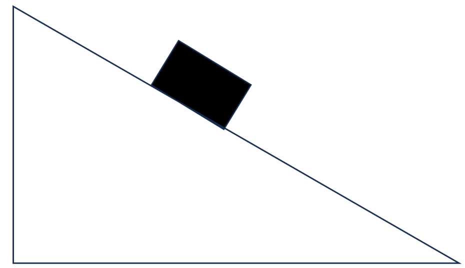
Prof. Laszlo Bardoczi, November 17, 2025

- The exam is 40 minutes long and contains 8 calculation problems (1-2 steps each).
- You must solve 6 problems of your choice; 2 problems will not be graded.
- Each problem is worth up to 1 point: 0.5 point for a correct numerical value (to 2 significant figures); 0.5 point for correct SI units.
- The concepts and calculations are broken into 1-2 step individual problems to award "partial credit". Such credits are based solely on the boxed answers.
- Write final answers clearly in the designated boxes. Leave the boxes empty for the problems you do not wish to be graded. If all boxes are filled, Problems #7-8 will not be graded.
- A detachable formula sheet is provided at the end of the exam.
- At the end of the exam, submit only the front page, present your UCI ID for verification and sign in.
- Direct all grading questions to your TA.
- Cheating will result in an automatic failing grade for the course.

Q#	Answer	Q#	Answer
1		2	
3		4	
5		6	
7		8	

Problem 1

A block of mass 3 kg rests on a slope inclined at 35°. A force of 6 N, parallel to the slope and directed downward, is applied. What is the minimum coefficient of static friction needed so that the block does not slip?



Solution:

For equilibrium, the net force along the incline must be zero:

$$f_s = mg\sin\theta + F$$

At the threshold of slipping, $f_s = \mu_s N = \mu_s mg\cos\theta$.

Equating:

$$\mu_s mg\cos\theta = mg\sin\theta + F$$

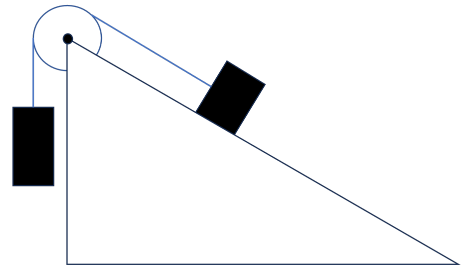
Solve for μ_s:

$$\mu_s = \frac{mg\sin\theta + F}{mg\cos\theta} = 0.95$$

Thus, the minimum coefficient of static friction required is $\mu_s = 0.95$.

Problem 2

A block on a 40° slope is connected by a light rope over a frictionless pulley to a second hanging block of the same mass. This system is located in an elevator going up at a constant speed of 6 m/s. What is the minimum coefficient of static friction required so that neither block moves?



Solution:

The hanging block tends to move *down*, pulling the slope block *up* the incline. Static friction on the slope block therefore acts *down* the incline, opposing that motion.

For equilibrium,

$$mg\sin\theta + \mu_s mg\cos\theta = mg$$

Simplify:

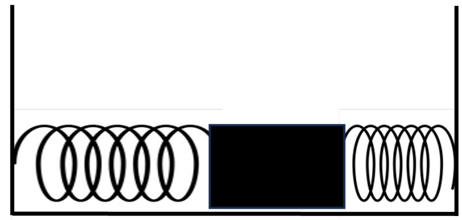
$$\sin\theta + \mu_s\cos\theta = 1$$

Substitute $\theta = 40°$:

$$\mu_s = \frac{1 - \sin 40°}{\cos 40°} = \frac{1 - 0.643}{0.766} = 0.47$$

Problem 3

A 1.5 kg block is attached between two identical horizontal springs, each with spring constant $k = 150\,\text{N/m}$, on a frictionless surface. The block is pulled 0.12 m to the right (positive direction) and held at rest. If static friction were present, what minimum static frictional force would be required to keep the block from sliding back toward equilibrium? State the magnitude, direction (sign), and units of the force.



Solution:

When the block is displaced 0.12 m to the right, both springs exert forces toward equilibrium:

- The **right spring** is stretched by $\Delta x_0 = 0.12\,\text{m}$ and pulls left with $F_R = -k\Delta x_0$.
- The **left spring** is compressed by $\Delta x_0 = 0.12\,\text{m}$ and also pushes left with $F_L = -k\Delta x_0$.

Thus, both forces act in the same (leftward) direction:

$$F_{\text{net}} = F_L + F_R = -2k\Delta x_0 = -36\,\text{N}$$

Therefore, a static frictional force of equal magnitude must oppose this motion to keep the block at rest:

$$F_s = 36\,\text{N}$$

Problem 4

A small sphere of mass $m = 1.2\,\text{kg}$ is dropped from rest through air. Its velocity as a function of time is given by

$$v(t) = v_0\big(1 - e^{-t/\tau}\big)\,,$$

where the terminal velocity is $v_0 = 4.9\,\text{m/s}$ and the time constant is $\tau = 0.5\,\text{s}$.

Find the absolute value of the acceleration at the moment it is released. State the magnitude and the unit.

Solution:

Differentiate the velocity:

$$a(t) = \frac{dv}{dt} = \frac{v_0}{\tau}e^{-t/\tau}.$$

At $t = 0$,

$$a(0) = \frac{v_0}{\tau} = \frac{4.9}{0.5} = 9.8\,\text{m/s}^2 \text{ (downward).}$$

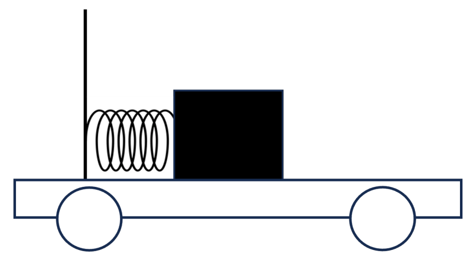
Alternatively, since the sphere is just released and air resistance is zero,

$$a(0) = g = 9.8\,\text{m/s}^2 \text{ (approx).}$$

Interpretation: At the instant of release, air resistance is negligible, so the sphere accelerates rapidly—its theoretical initial acceleration equals v_0/τ, here $15\,\text{m/s}^2$ downward.

Problem 5

A block of mass $m = 1\,\text{kg}$ rests on the flat, frictionless surface of a cart, supported by a horizontal spring of constant $k = 25\,\text{N/cm}$ on the left side. The cart begins to accelerate horizontally at a constant rate of $a = 4\,\text{m/s}^2$ to the right (positive direction). What is the compression of the spring? Specify the direction (plus or minus), magnitude and unit.



Solution:

In the accelerating frame of the cart, the block experiences a fictitious force opposite the direction of acceleration:

$$F_{\text{fict}} = -ma.$$

At equilibrium (no relative motion between block and cart), the spring's restoring force balances this fictitious force:

$$-kx - ma = 0.$$

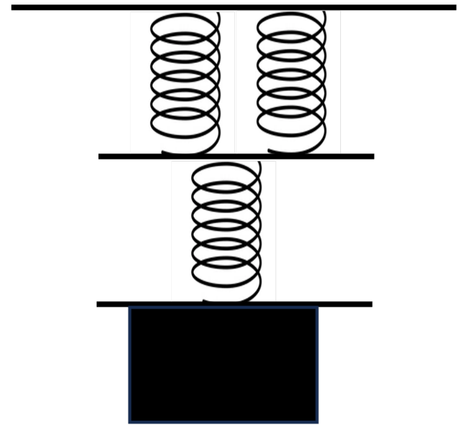
Thus, the compression of the spring is:

$$x = -\frac{ma}{k} = -1.6\,\text{mm}\,\text{mm (in the direction opposite the acceleration).}$$

Interpretation: As the cart accelerates, the block lags slightly, compressing the spring until its restoring force equals the inertial (fictitious) force.

Problem 6

A box of mass $m = 3.0\,\text{kg}$ hangs from a system of three identical vertical springs, each with spring constant $k = 800\,\text{N/m}$. Two of the springs are connected in parallel, and this parallel combination is attached in series with a third spring, as shown. Assuming the springs and connecting planks are massless, determine the total extension of the spring system when the box is in static equilibrium. State the magnitude and the unit. Take the positive direction to be downward.



Solution:

The two parallel springs share the same extension and have an equivalent spring constant:

$$k_p = k + k = 2k = 1600\,\text{N/m}.$$

This parallel pair is connected in series with the third spring, so the overall equivalent spring constant is:

$$\frac{1}{k_{\text{eq}}} = \frac{1}{k_p} + \frac{1}{k} \longrightarrow k_{\text{eq}} = 2k/3.$$

At equilibrium, the net upward spring force equals the weight of the hanging box:

$$k_{\text{eq}}\,x = mg \longrightarrow x = 5.5\,\text{cm}.$$

Problem 7

The human jaw acts as a lever system. Suppose the jaw muscle (masseter) attaches to the mandible at a point 2.5 cm from the jaw hinge, and the bite force is exerted by the teeth at a point 7.0 cm from the hinge. If the muscle exerts an upward force of $F_m = 600\,\mathrm{N}$ (in the positive direction), what is the bite force F_b at the teeth, assuming static equilibrium? State the direction (plus or minus), magnitude and unit.

Solution:

The jaw rotates about the hinge. In equilibrium, the net torque about the hinge is zero:

$$\tau_{\text{muscle}} = \tau_{\text{bite}}.$$

Each torque is the product of force and its lever arm:

$$F_m r_m = F_b r_b.$$

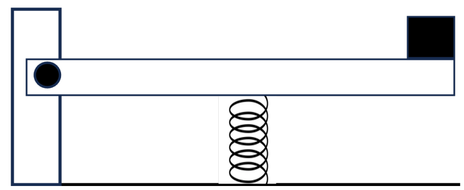
Solve for the bite force:

$$F_b = \frac{r_m}{r_b} F_m = 214\,\mathrm{N}.$$

Interpretation: Because the teeth are farther from the hinge than the muscle attachment, the jaw provides a *mechanical disadvantage*: a large muscle force produces a smaller bite force. In carnivores, the muscle attaches farther from the hinge, increasing leverage and bite strength.

Problem 8

A massless plank of length $L = 2.0\,\text{m}$ is hinged at its left end and supported halfway along its length (at $L/2$) by a vertical spring of constant $k = 3500\,\text{N/m}$. The hinge is located $0.40\,\text{m}$ above the ground, and the spring's natural (unstretched) length is $L_0 = 0.50\,\text{m}$. A small block is placed at the right end of the plank and presses downward with a force equal to its weight. If the plank is to remain perfectly horizontal in static equilibrium, what downward force F must the block apply at the right end? State the magnitude and the unit.



Solution:

When the plank is perfectly horizontal, the height of the spring attachment point (at $L/2$) is the same as the hinge height, $0.40\,\text{m}$. The spring's natural length is $L_0 = 0.50\,\text{m}$, so its compression is:

$$\Delta x = L_0 - 0.40 = 0.10\,\text{m}.$$

The upward spring force is then:

$$F_s = k\,\Delta x = (3500)(0.10) = 350\,\text{N}.$$

Taking torques about the hinge for static equilibrium:

$$F_s\left(\frac{L}{2}\right) = F(L) \quad \Longrightarrow \quad F = \frac{F_s}{2}.$$

Substitute $F_s = 350\,\text{N}$:

$$F = \frac{350}{2} = 175\,\text{N}.$$

Interpretation: To keep the plank horizontal, the block must exert a downward force of $175\,\text{N}$ at the right end, balancing the torque produced by the spring's $350\,\text{N}$ upward force at the midpoint.

Physics 3A Formula Sheet

Vectors

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$
$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$
$$A_x = |\vec{A}|\cos\theta$$
$$A_y = |\vec{A}|\sin\theta$$
$$\tan\theta = \frac{A_y}{A_x}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j}$$
$$\vec{A} + \vec{B} = (A_x + B_x, A_y + B_y)$$
$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = |\vec{A}||\vec{B}|\cos\phi$$

Trigonometry

$$\sin\theta = \cos(90^\circ - \theta)$$
$$\cos\theta = \sin(90^\circ - \theta)$$
$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$F_x = |\vec{F}|\cos\theta$$
$$F_y = |\vec{F}|\sin\theta$$

Geometry Essentials

$$c^2 = a^2 + b^2$$
$$C = 2\pi r; A = \pi r^2; s = r\theta$$
$$A = 4\pi r^2; V = \frac{4}{3}\pi r^3$$

Diffusion and Proportional Reasoning

$$r_{\text{rms}} = d\sqrt{mn}$$
$$r_{\text{rms}}^2 = d^2mn$$
$$D = \tfrac{1}{2}vd$$
$$r_{\text{rms}} = \sqrt{2mDt}$$

$$y = Cx^n$$
$$\frac{y_2}{y_1} = \frac{Cx_2^n}{Cx_1^n}$$

Kinematics

$$\Delta x = v\Delta t$$
$$v_{\text{avg.}} = \Delta x/\Delta t$$
$$a = \Delta v/\Delta t$$
$$x(t) = x_0 + vt$$
$$v(t) = v_0 + at$$
$$x(t) = x_0 + v_0t + \frac{1}{2}at^2$$
$$x(t) = \int v(t)\,dt$$
$$v(t) = \int a(t)\,dt$$

$$v(t) = \frac{dx}{dt}$$
$$a(t) = \frac{dv}{dt}$$
$$\frac{d}{dt}[t^n] = nt^{n-1}$$
$$\int_{t_1}^{t_2} t^n\,dt = \frac{t_2^{n+1} - t_1^{n+1}}{n+1}$$
$$y(t) = v_{y0}t - \frac{1}{2}gt^2$$

$$t_{\text{flight}} = \frac{2v_{y0}}{g}$$
$$y_{\text{max}} = \frac{v_{y0}^2}{2g}$$
$$x_{\text{final}} = \frac{2v_{x0}v_{y0}}{g}$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x_{\text{end}} = \frac{v_{x0}}{g}\left(v_{y0} + \sqrt{v_{y0}^2 - 2gy_{\text{end}}}\right)$$
$$t_{\text{flight}} = \frac{v_{y0} + \sqrt{v_{y0}^2 - 2gy_{\text{end}}}}{g}$$
$$v_y = -\sqrt{v_{y0}^2 - 2gy_{\text{end}}}$$

Interacting Systems

$$F_g = mg$$
$$F_s = -k\,\Delta x$$
$$K = k_1 + k_2$$
$$\frac{1}{K} = \frac{1}{k_1} + \frac{1}{k_2}$$
$$F_T = \text{constant along the rope}$$
$$F_N = mg\cos\theta$$
$$F_{\mu,s} \leq \mu_s N$$
$$F_{\mu,k} = \mu_k N$$
$$F_D = -\tfrac{1}{2}C_d\,\rho A v^2$$
$$F_D = -6\pi\,\eta\,r\,v$$
$$F_{\text{thrust}} = \dot{m}\,v_{\text{exhaust}}$$
$$a = \frac{F_{\text{net}}}{m}$$

Torque

$$\tau = rF\sin(\theta)$$
$$\alpha = \frac{\tau_{\text{net}}}{I}$$