Physics 3A – Midterm 3 Review

Prof. Laszlo Bardoczi

Torque and Rotational Dynamics

$$\tau = rF\sin\theta$$

Concepts:

- Torque measures how effectively a force causes rotation.
- Depends on lever arm r and angle θ between r and F. In most of our applications, the arm and the force are perpendicular to each other, making $\theta = 90^\circ$ and hence $\sin(\theta) = 1$, leaving $\tau = rF$

$$\tau = I\alpha$$

Concepts:

- Rotational version of $F = ma$.
- I (moment of inertia) $=$ resistance to rotational acceleration.

Torque Applications from Lecture Slides

- **Child pushing a merry-go-round**
  Tangential force creates torque: $\tau = FR$. (p3a_lec_7.2.pdf, slide 7)
- **Two children on a seesaw**
  Opposing torques: $\tau_{\text{net}} = m_2 g l_2 - m_1 g l_1$. (p3a_lec_7.2.pdf, slide 8)
- **Seesaw imbalance determining rotation direction**
  Compare left vs right torques. (p3a_lec_7.2.pdf, slide 16)
- **Rod released from the horizontal**
  Gravity produces torque: $\tau = mg\frac{L}{2}$. (p3a_lec_7.2.pdf, slide 10)
- **Bucket pulling a rotating disk**
  Rope tension causes torque: $\tau = TR$. (p3a_lec_7.2.pdf, slide 12)
- **Yo-yo unwinding while falling**
  Torque from tension drives rotation: $TR = I\alpha$. (p3a_lec_7.2.pdf, slide 15)
- **Rod–spool system driven by falling mass**
  Rope tension applies torque on the spool to rotate the system.
  (p3a_lec_7.2.pdf, slide 13)

Circular Motion & Angular Quantities (1/2)

Period and Frequency

$$f = \frac{1}{T}$$

- Period T = time for one revolution.
- Frequency f = number of revolutions per second.

Speed in Circular Motion

$$v = \frac{2\pi r}{T} = 2\pi r f$$

- Speed depends on radius and rate of rotation.
- Example: edge speed on a centrifuge rotor.

Angular Speed

$$\omega = 2\pi f$$

- Angular speed: radians per second.
- Links linear motion to rotational motion.

Angular Kinematics

$$\begin{aligned} \phi &= \omega t \quad \text{and} \quad \omega = \text{const.} \quad \text{(uniform rotation)} \\ \phi &= \omega_0 t + \tfrac{1}{2}\alpha t^2 \quad \text{(accelerating rotation)} \\ \omega &= \omega_0 + \alpha t \quad \text{(accelerating rotation)} \end{aligned}$$

- Rotational analogs of constant-acceleration equations.

Examples from Lecture Slides (1/2)

Period & Frequency

- Car on a circular track, London Eye, spinning bicycle wheel (p3a_lec_7.1.pdf, slide 1)
- Table saw blade (compute T and f) (p3a_lec_7.1.pdf, slide 3)
- Quasar carnival ride – finding rotation period (p3a_lec_7.1.pdf, slide 10)

Speed in Circular Motion ($v = 2\pi rf$)

- Table saw blade – speed of a tooth at the rim (p3a_lec_7.1.pdf, slide 3)
- Quasar carnival ride – riders' speed using $v = \sqrt{ar}$ (p3a_lec_7.1.pdf, slide 10)

Angular Speed ($\omega = 2\pi f$)

- Circular motion coordinates: $x(t) = r\cos(\omega t)$, $y(t) = r\sin(\omega t)$ (p3a_lec_7.1.pdf, slide 8)
- Table saw blade – rpm $\rightarrow$ frequency $\rightarrow$ angular speed (p3a_lec_7.1.pdf, slide 3)

Examples from Lecture Slides (2/2)

Angular Kinematics ($\omega = \omega_0 + \alpha t$, $\phi = \omega t$, ...)

- Merry-go-round pushed by a child – torque $\rightarrow \alpha \rightarrow$ final ω (p3a_lec_7.2.pdf, slide 7)
- Rod released from horizontal – gravity torque gives angular acceleration (p3a_lec_7.2.pdf, slide 10)
- Yoyo unwinding – tension creates α leading to changing $\omega(t)$ (p3a_lec_7.2.pdf, slide 15)

What is the Centripetal Force?

The centripetal force is not a new or mystical force.

It is simply the **net force that makes an object follow a circular path**. Even if the **speed stays constant**, the velocity is changing direction, and a change in velocity requires a force. The centripetal force is the inward force that provides the needed acceleration

$$a = \frac{v^2}{r}.$$

Which real force acts as the centripetal force depends on the situation:

- **Tension** (Tarzan swinging on a vine)
- **Normal force** (rollercoaster at the bottom of a loop)
- **Gravity** (rollercoaster at the top of a loop)
- **Friction** (car turning on a road)

Key idea: The centripetal force is the **net inward force from the free-body diagram**. It **changes the direction** of the velocity (not its magnitude) and always points **toward the center** of the circular path.

Moments of Inertia

Concepts:

- Rotational inertia depends on mass distribution.
- More mass farther from the axis $\rightarrow$ larger I.
- Moments of inertia are listed on the equation sheet.
  You just need to recognize which one is which.

Momentum Basics

$$p = mv \quad \text{and} \quad \frac{dp}{dt} = F(t)$$

- Momentum measures "quantity of motion."
- Force changes momentum over time.

$$p_f - p_i = \int F(t)dt = J$$

- Impulse (J) = change in momentum.
- Longer contact time $\rightarrow$ smaller force needed (e.g., padded shoes).

$$P_{\text{tot}} = p_1 + p_2 \quad \text{and} \quad \Delta p_1 + \Delta p_2 = 0$$

- Internal forces in an isolated system cancel due to Newton's third law $\rightarrow$ total momentum stays constant.

Examples from Lecture Slides: Momentum Concepts

Momentum

- Golf club striking a ball — large, short force changes momentum (p3a_lec_8.1.pdf, slide 1)
- Force–time graph during a collision — momentum change equals area under $F(t)$ (p3a_lec_8.1.pdf, slide 5)

Impulse

- Collision force modeled as a triangle — computing F_{max} from impulse (p3a_lec_8.1.pdf, slide 11)
- Falling object impact — impulse relates average impact force and fall height (p3a_lec_8.1.pdf, slide 13)

Momentum of a System

- Two blocks stuck together move with same v (p3a_lec_8.1.pdf, slide 14)

Momentum Conservation ($\Delta p_1 + \Delta p_2 = 0$)

- Collision of two balls — Newton's 3rd law $\Rightarrow$ equal and opposite momentum change (p3a_lec_8.1.pdf, slide 15)
- Example: Two colliding train cars stick together (p3a_lec_8.1.pdf, slide 17)

Energy

- Kinetic energy (K) measures the energy of motion; potential energies (U) store energy based on position (height or spring compression).
- Total mechanical energy is conserved when no non-conservative forces (friction, drag) act on the system.
- Changes in energy tell you what the system *can* do—how fast it will move or how high it can rise.

$$K = \frac{1}{2}mv^2$$
$$U = mgh$$
$$U = \frac{1}{2}kx^2$$
$$E = K + U$$

Work

- Work is the process of transferring energy into or out of a system by applying a force over a distance.
- Only the component of force along the direction of motion contributes to work (recall properties of the dot product).
- Positive work increases the system's mechanical energy; negative work (like friction) removes mechanical energy and turns it into heat (energy, in general, is conserved).

$$W = Fx$$
$$W = \int F(x)\,dx$$
$$W = \Delta K$$
$$W = \Delta E$$

Power

- Power measures how fast work is done or how quickly energy is transferred.
- A large force doesn't guarantee high power—both force and speed matter.
- Human muscles have a maximum power output, which limits sprint speed and lifting performance.

$$P = \frac{dW}{dt} = Fv$$

Examples from Lecture Slides: Energy Concepts (1/2)

Kinetic and Potential Energy ($K = \frac{1}{2}mv^2$, $U = mgh$, $U = \frac{1}{2}kx^2$)

- Free–falling ball: gravitational potential → kinetic (p3a_lec_10.1.pdf, slide 11)
- Block sliding down frictionless slope — conversion of U_g to K (p3a_lec_10.1.pdf, slide 11–12)
- Hooke's law spring compression — stored spring potential energy (p3a_lec_10.1.pdf, slide 6)

Total Mechanical Energy ($E = K + U$)

- Conservative systems: ball in free fall, mass on a spring — energy exchanges between K and U (p3a_lec_10.1.pdf, slide 10)
- Examples list: free fall, frictionless slope, simple harmonic oscillators (p3a_lec_10.1.pdf, slide 10)

Examples from Lecture Slides: Energy Concepts (2/2)

Work ($W = Fx$, $W = \int F(x)dx$, $W = \Delta K$, $W = \Delta E$)

- Work by gravity during vertical motion (p3a_lec_10.1.pdf, slide 6)
- Work done by friction (negative work, energy lost to heat) (p3a_lec_10.1.pdf, slide 6)
- Work done compressing a spring (requires integral) (p3a_lec_10.1.pdf, slide 6)
- Work–energy theorem ($W = \Delta K$) derived from $\int v\, dp$ (p3a_lec_10.1.pdf, slide 7)

Power ($P = dW/dt = Fv$)

- Power in everyday activities: muscles doing work at a rate (see textbook)